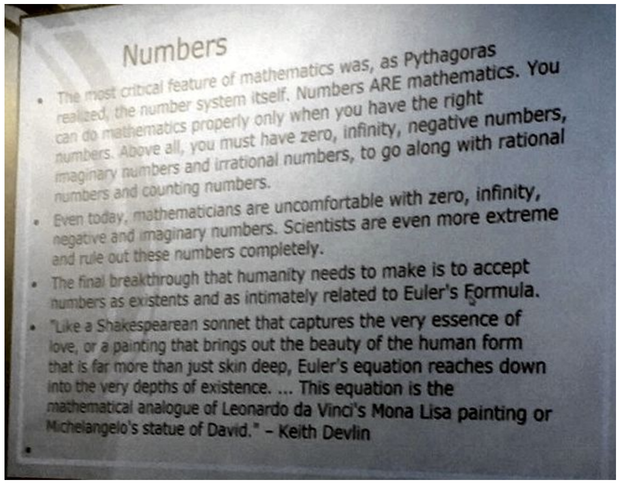
MATH 106: History of Mathematics



The photograph is a slide presented in a History of Mathematics course that a colleague of mine took as an undergrad at UConn. The professor plagiarized the slide from a book called *The God Equation* by Mike Hockney—which is the collective pseudonym of a group of (crackpot) conspiracy theorists. *Our* course will be more factual (and perhaps proportionally less entertaining, though that's probably for the best).

“THEORETICAL MATHEMATICS”: TOWARD A CULTURAL SYNTHESIS OF MATHEMATICS AND THEORETICAL PHYSICS

“**Abstract**. Is speculative mathematics dangerous? Recent interactions between physics and mathematics pose the question with some force: traditional mathematical norms discourage speculation, but it is the fabric of theoretical physics. In practice there can be benefits, but there can also be unpleasant and destructive consequences. Serious caution is required, and the issue should be considered before, rather than after, obvious damage occurs. With the hazards carefully in mind, we propose a framework that should allow a healthy and positive role for speculation.”

—A. Jaffe and F. Quinn

https://arxiv.org/pdf/math/9307227.pdf

Brief Summary

- Modern mathematics is characterized by the use of *proofs* (rigorous, logical arguments based on a set of accepted first principles, i.e., axioms)
- In physics, theories are tested and supported by empirical evidence. JQ claim there is a parallel in mathematics, where conjectural results are tested by proof/disproof
- JQ note there is a division of labor in physics between people who propose theories and people who find empirical evidence for/against them: theoretical physics vs. experimental physics
- JQ claim there has been a recent tendency amongst some mathematicians to theorize, speculate, and argue heuristically/intuitively without proof—thereby occupying the same intellectual niche in mathematics as theoretical physicists occupy in physics. JQ call this *theoretical mathematics*

Brief Summary (cont.)

- JQ present some examples of theoretical mathematics, split between "success stories" and "cautionary tales." From the latter they formulate a list of problems with theoretical mathematics:
  1. Theoretical work lacks the feedback/correction provided by rigorous proof
  2. Further work is discouraged by uncertainty about what is reliable
  3. A dead area is created when too much credit is claimed by theorizers
  4. Students and young researchers are misled
- JQ acknowledge that theoretical mathematics can be useful to the community, and go on to prescribe some best practices to adopt:
  1. Majority of credit for completing/proving theoretical work should go to the prover, not the theorizer
  2. A clear vocabulary should be adopted
  3. Only fully proven results should be announced

The Comparison to Physics

"Modern mathematics is nearly characterized by the use of rigorous proofs. This practice, the result of literally thousands of years of refinement, has brought to mathematics a clarity and reliability unmatched by any other science. But it also makes mathematics slow and difficult; it is arguably the most disciplined of human intellectual activities." —JQ (P.1)

"Theoretical physics and mathematical physics have rather different cultures, and there is often a tension between them. Theoretical work in physics does not need to contain verification or proof, as contact with reality can be left to experiment. Thus the sociology of physics tends to denigrate proof as an unnecessary part of the theoretical process. Richard Feynman used to delight in teasing mathematicians about their reluctance to use methods that "worked" but that could not be rigorously justified [F, G2]. He felt it was quite satisfactory to test mathematical statements by verifying a few well-chosen cases." —JQ (P.5)

The Comparison to Physics (cont.)

“Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap…

“In the middle of the twentieth century it was attempted to divide physics and mathematics. The consequences turned out to be catastrophic. Whole generations of mathematicians grew up without knowing half of their science and, of course, in total ignorance of any other sciences…

Attempts to create "pure" deductive-axiomatic mathematics have led to the rejection of the scheme used in physics (observation - model - investigation of the model - conclusions - testing by observations) and its substitution by the scheme: definition - theorem - proof.”
—V. I. Arnold, *On Teaching Mathematics*

Arnold’s full essay is here: https://www.uni-muenster.de/Physik.TP/~munsteg/arnold.html. I should warn that the content of this essay is even more contentious than that of the JQ article, but it is an interesting cultural read nonetheless. I highly recommend it—especially to those who think they might someday teach higher-level mathematics.

The Comparison to Physics (cont.)

"For other and deeper reasons I cannot share the enthusiasm of Jaffe-Quinn for physics. Their comparison of proofs in mathematics with experiments in physics is clearly faulty. Experiments may check up on a theory, but they may not be final; they depend on instrumentation, and they may even be fudged. The proof that there are infinitely many primes—and also in suitable infinite progressions—is always there. We need not sell mathematics short, not even to please the ghost of Feynmann [sic]…

"Physics has provided mathematics with many fine suggestions and new initiatives, but mathematics does not need to copy the style of experimental physics. Mathematics rests on proof—and proof is eternal."—Saunders Mac Lane, in response to JQ

The Limits of Proof and Rigor

"I do still believe that rigor is a relative notion, not an absolute one. It depends on the background readers have and are expected to use in their judgment. Since the collapse of Hilbert's program and the advent of Gödel's theorem, we know that rigor can be no more than a local and sociological criterion." —René Thom, in response to JQ

Gödel's theorem roughly states that there is no set of axioms under which all true statements about natural numbers can be proven.

Corollary: There are mathematical statements that are true, yet unprovable.

As I mentioned in discussion, Gödel's theorem presents an epistemological problem to mathematicians not dissimilar to the the one presented to physicists by the Heisenberg uncertainty principle. In short, there are limits to what we can know with absolute certainty, and there are some mathematical statements that we may have to accept as merely "probably true" (much like how scientists accept some physical laws as probably true despite a highly limited ability to experimentally test them). To what degree should we allow this to guide (or stifle) avenues of research?

The Issue of Credit

"I have the impression that applying rigor to a theoretical idea is given substantial credit when it disconfirms the theoretical idea or when the proof is especially difficult or when the ideas of the proof are original, interesting and fruitful. This seems quite enough to motivate the application of rigor, for those who are motivated by the prospect of credit. Perhaps pedestrian proofs do get only a little recognition, but should they really get more? Is it useful to formulate explicit general rules for assigning credit in mathematics?"—Daniel Friedan, in response to JQ

A related example that I brought up in discussion was Mochizuki's purported proof the abc conjecture. A nice discussion of the proof and the debate surrounding it (including many mathematicians' reluctance to read it) can be found here: https://inference-review.com/article/a-crisis-of-identification

Research Announcements

In what year was the JQ article published? What is different today?

dat internet tho
such arxiv
much communication