(A) Propositional axiom schemes, same as in PL.

(1) $\phi \to (\psi \to \phi)$
(2) $(\phi \to \psi) \to ((\phi \to (\psi \to \chi)) \to (\phi \to \chi))$
(3) $(\phi \to \psi) \to ((\phi \to \neg\psi) \to \neg\phi)$
(4) $\neg\neg\phi \to \phi$
(5) $\phi \to (\psi \to (\phi \land \psi))$
(6a) $(\phi \land \psi) \to \phi$ (6b) $(\phi \land \psi) \to \psi$
(7a) $\phi \to (\phi \lor \psi)$ (7b) $\psi \to (\phi \lor \psi)$
(8) $(\phi \to \chi) \to ((\psi \to \chi) \to ((\phi \lor \psi) \to \chi))$

(B) Predicate axiom schemes.

(9) $\forall v\phi(v, \vec{u}) \to \phi(t, \vec{u})$ (t free for v in $\phi(v, \vec{u})$)
(10) $\forall v(\phi \to \psi) \to (\phi \to \forall v\psi)$ (v not free in ϕ)
(11) $\phi(t, \vec{u}) \to \exists v\phi(v, \vec{u})$ (t free for v in $\phi(v, \vec{u})$)

(C) Rules of inference.

(12) From ϕ and $\phi \to \psi$, infer ψ. (Modus Ponens)
(13) From ϕ, infer $\forall v\phi$. (Generalization)
(14) From $\phi \to \psi$, infer $\exists v\phi \to \psi$, provided v is not free in ψ. (Exists Elimination)

(D) Identity axioms. Skipped.

Lemma 7C.4. The natural introduction rules for LPCI.
If T is a set of sentences, the indicated substitutions are free, and the indicated restrictions are obeyed, then the following hold:

($\to$) *If $T, \chi \vdash \phi$, then $T \vdash \chi \to \phi$.* *Restriction: χ is a sentence.*
($\land$) *If $T \vdash \phi$ and $T \vdash \psi$, then $T \vdash \phi \land \psi$.*
($\lor$) *If $T \vdash \phi$ or $T \vdash \psi$, then $T \vdash \phi \lor \psi$.*
($\neg$) *If $T, \chi \vdash \psi$ and $T, \chi \vdash \neg\psi$, then $T \vdash \neg\chi$.* *Restriction: χ a sentence.*
($\forall$) *If $T \vdash \phi$, then $T \vdash \forall v\phi$.*
($\exists$) *If $T \vdash \phi\{v :\equiv t\}$, then $T \vdash \exists v\phi$.*

Lemma 7C.5. The natural elimination rules for LPCI.
If T is a set of sentences, the indicated substitutions are free, and the indicated restrictions are obeyed, then the following hold:

($\to$) *If $T \vdash \phi$ and $T \vdash \phi \to \psi$, then $T \vdash \psi$.*
($\land$) *If $T \vdash \phi \land \psi$, then $T \vdash \phi$ and $T \vdash \psi$.*
($\lor$) *If $T, \phi \vdash \chi$ and $T, \psi \vdash \chi$, then $T, \phi \lor \psi \vdash \chi$*
Restriction: ϕ, ψ are sentences.
($\neg$) *If $T \vdash \neg\neg\phi$, then $T \vdash \phi$.*
($\forall$) *If $T \vdash \forall v\phi$, then $T \vdash \phi\{v :\equiv t\}$*
($\exists$) *If $T, \phi\{v :\equiv c\} \vdash \chi$, then $T, \exists v\phi \vdash \chi$*
Restriction: $\exists v\phi$ is a sentence and c is a constant which does not occur in T or in χ.