Please show **all** your work! Answers without supporting work will not be given credit.

Name:____

1. **Logistics**
   - Office Hours: MS 3957, Tuesday at 10a and Thursdays at 11a
   - SMC: MS 3974 Monday-Thursday 9a-3p (I'll be there Thursdays at 10a)

2. **And We're Free, Free Fallin'**

   With Calculus, we're able to solve problems about motion, such as free fall. In these questions, we are typically given information such as acceleration as a function of time, initial velocity, and initial position. The first part of the strategy is to anti-differentiate $a(t)$ to get $v(t)$ and $s(t)$. For example if we are in free fall on Mars, then $a(t) = -3.8$ m/s^2. Suppose our initial velocity is 0 m/s and our initial position is 100 m above the surface of Mars, then a natural first question may be, at what time do we reach the surface? First we get $v(t) = -3.8t$, and then $s(t) = -1.9t^2 + 100$. Reaching the surface corresponds to $s(t) = 0$, and we only take the positive answer $t \approx 7.25$ seconds.

   Your turn:

   (a) Suppose you drop an iPhone off the top of the Burj Khalifa (height: 828m). What will the velocity be when the iPhone hits the ground?
   $a(t) = -9.8$ and so $v(t) = -9.8t + v(0)$ but since you're just dropping it, it has no inital velocity, so $v(t) = -9.8t$. Thus $s(t) = -4.9t^2 + 828$. To find out when it hits the ground, solve $s(t) = 0$ to get $t = 13$ and our answer is $v(13) = 127$ meters/second.

   (b) Suppose you're trying to dunk on the moon where the acceleration due to gravity is -1.67 m/s^2. A person typically needs to jump about 0.9 m off the ground to dunk. What will your initial jump velocity need to be?
   $a(t) = -1.67$ and so $v(t) = -1.67t + v_0$. Our goal is to find v_0 such that $s(t) = -1.67t^2/2 + v_0 t$ is at least 0.9 somewhere. Let's check for worst case where you still make it, i.e. your maximum height is 0.9. The maximum occurs at time $t^* = v_0/1.67$ and so $s(t^*) = -1.67(v_0/1.67)^2/2 + v_0(v_0/1.67) = v_0^2/(2(1.67))$ is the maximum height. We want this to be at least 0.9, so $v_0 \geq \sqrt{2(1.67)(0.9)}$

3. **America's Next Top Differential Model** Modeling and Differential Equations are two huge fields of math in and of themselves. Our attention will be focused on formulating modeling problems as differential equations and introducing some basic techniques used to solve differential equation. So what is differential about differential equations? Instead of questions like, 'for what values of x is the equation $x^3 + \sin(x) = 1 + x^x$ true?', we ask questions like 'for what functions $f(x)$ is the differential equation $f'(x) = 4f(x) + x$ true?' or 'for what functions y is the differential equation $\frac{dy}{dt} = ky$ true?' Differential equations show up in modeling because often the quantities we want to study and their rates of change are related, which gives rise to differential equations.

   Write a differential equation to model the following situations:

   (a) The number of golf trips Donald Trump has taken grows at an approximately constant rate of 0.18 trips per day.
   $\frac{dN}{dt} = 0.18$

   (b) Newton's second law says that the mass of an objects multiplied by the acceleration (second derivative of position) of the object is equal to the force F on the object.
   $\frac{d^2x}{dt^2}m = F$

(c) The rate at which an epidemic spreads through a community of P susceptible people is proportional to the product of the number of people who have caught the disease, y and the number who have not, $P - y$.
$\frac{dy}{dt} = ky(P - y)$

4. **Prepare for trouble, make it double**

Find the time for the population to double or halve its initial level, as appropriate:

(a) $\frac{dN}{dt} = N$

$N = Be^t$ and so the doubling time is found by solving $Be^t = 2B$ which gives $\ln(2)$.

(b) $\frac{dN}{dt} = 2N$
$N = Be^{2t}$ and so the doubling time is found by solving $Be^{2t} = 2B$ which gives $\ln(2)/2$.

(c) $\frac{dN}{dt} = 0.5N$
$N = Be^{.5t}$ and so the doubling time is found by solving $Be^{.5t} = 2B$ which gives $2\ln(2)$.

(d) $\frac{dN}{dt} = -N$
$N = Be^{-t}$ and so the half-life time is found by solving $Be^{-t} = B/2$ which gives $\ln(2)$.

5. **King Arthur's Round Table** [1]

In Winchester castle there hangs a wooden round table, 18 feet in diameter and divided into twenty-five sections, one for the king and twenty-four for the knights. Some speculate that the Winchester round table is King Arthur's round table from the fifth century. We know that the round table has been at Winchester since the fifteenth century. John Harding says in his chronical (1484) that the round table ended at Winchester, and there it hangs still. To put an end to the speculation regarding the Winchester round table, in 1976 it was taken down from the wall and tests were employed to determine the date of origin. The rate of decay of carbon-14 in the table (i.e., in dead wood) was found to be 6.08 atoms per minute per gram of sample. Estimate the age of the table to determine whether the Winchester table was King Arthur's round table. Hint: Use the facts that the half-life of carbon-14 in dead wood is 5,568 years and that in living wood the rate of decay of carbon-14 is 6.68 atoms per minute per gram of wood.

We use the rate of carbon-14 decaying in order to determine how much carbon-14 is left compared to the starting amount. That is, the table is decaying only 6.08 atoms per minute, but living wood (presumably the same volume of living wood) decays at a rate of 6.68, so the table has 0.91 as much carbon-14 as it started with. Note, to make this conclusion we are using the fact that decay rate is proportional to amount of carbon-14 in a substance. So we must solve $N(t) = N_0 e^{-\lambda t} = 0.91 N_0$ knowing that the decay constant $\lambda = \ln(2)/5568 = 0.00012448764$ where 5568 is the half-life measured in years. Thus we solve for $t = 756$ years and so the table probably is not from the fifth century unless they somehow kept the tree alive while using it as a table.

[1] Question 6.1.33 Schreiber, Calculus for Life Sciences