Math 3B Week 5 Discussion Practice Problems Solutions May 30, 2017

Please show **all** your work! Answers without supporting work will not be given credit.

Name:____

1. **Logistics**
   - Office Hours: MS 3957, Tuesday at 10a and Thursdays at 11a
   - SMC: MS 3974 Monday-Thursday 9a-3p (I'll be there Thursdays at 10a)

2. **Definite Integrals as Sums**

   Express the following as limits of Riemann Sums:

   (a) $\displaystyle\int_{-1}^{1} (x^2 - x)\,dx = \lim_{n\to\infty} \sum_{i=0}^{n} \left((-1 + \frac{2}{n})^2 - (-1 + \frac{2}{n}) \right) \frac{2}{n}$

   (b) $\displaystyle\int_{-1}^{1} |x|\,dx = \lim_{n\to\infty} \sum_{i=0}^{n} \left(|-1 + \frac{2}{n}| \right) \frac{2}{n}$

   (c) $\displaystyle\int_{-1}^{1} |\cos(x)|\,dx = \lim_{n\to\infty} \sum_{i=0}^{n} \left(|\cos(-1 + \frac{2}{n})| \right) \frac{2}{n}$

3. **What did the acorn say when he grew up?**

   Use geometry to evaluate the following definite integrals (remember that the area is signed, i.e. area below the x-axis is negative):

   (a) $\displaystyle\int_{-1}^{1} |x|\,dx = 1$

   There are two triangles, each of area 1/2.

   (b) $\displaystyle\int_{0}^{10} \sqrt{25 - (x-5)^2} + 5\,dx = 50 + \frac{25\pi}{2}$

   This is the upper half of a circle centered at (5,5) with radius 5. So in integrating we have the semicircle which contributes $\frac{25\pi}{2}$ and then the rectangle of width ten and height five.

   (c) $\displaystyle\int_{-1}^{4} x\,dx = 7.5$

   There are two triangles, one above the x axis, that has area 8 and one below the x axis which has area $\frac{1}{2}$.

4. **Just one more episode...**

   Assume that over an 11 week period, the rate at which you binge watch Netflix is given by

$$f(x) = \begin{cases} 28 & 0 \le x \le 3 \\ 31 - x & 3 < x \le 9 \\ 40 & 9 < x \le 10 \\ 10x - 100 & 10 < x \le 11 \end{cases}$$

   where x is weeks and $f(x)$ is hours watched per week. How many hours have you watched after the 11 weeks?

   Integrating this function piecewise gives

$$\int_0^{11} f(x)dx = \int_0^3 28dx + \int_3^9 31 - xdx + \int_9^{10} 40dx + \int_{10}^{11} 10x - 100dx =$$

$$84 + 150 + 40 +$$

5. **The Fun(damental) Theorem of Calculus**

   Evaluate the following definite integrals:

   (a) $\int_{-1}^{1} e^x \, dx = e^x \Big|_{-1}^{1} = e - \frac{1}{e}$

   (b) $\int_0^1 x^{2017} \, dx = \frac{x^{2018}}{2018} \Big|_0^1 = \frac{1}{2018}$

   (c) $\int_0^{\pi/2} \cos(x) \, dx = \sin(x) \Big|_0^{\pi/2} = 1$

The End.