

Please show **all** your work! Answers without supporting work will not be given credit.

Name: _____

1. Logistics

- Office Hours: MS 3957, Tuesday at 10a and Thursdays at 11a
- SMC: MS 3974 Monday-Thursday 9a-3p (I'll be there Thursdays at 10a)

2. **Rapid Fire Antiderivatives** We've learned the techniques of differentiation, now we practice undoing it. For each differentiation method there's an analogous integration method. For example, to differentiate x^n , we apply the power rule to get nx^{n-1} . Now in reverse, to anti-differentiate x^m we increment the exponent up by one and multiply the whole thing by the reciprocal of this new exponent to get $\frac{1}{m+1}x^{m+1}$. This isn't the general antiderivative though. The general one is $\frac{1}{m+1}x^{m+1} + C$ because if we were to differentiate this, the constant would disappear, so we have to account for that possibility.

Now let's practice, find the general anti-derivative of the following:

- (a) $10x^5 + 3x^2 + 5x$
- (b) $\frac{3}{x^2}$
- (c) $3\sin(\pi x)$

If we are given more information, sometimes called initial conditions or boundary conditions, we can solve for C . For example if we want to find the anti-derivative $F(x)$ of the function $f(x) = x^3$ with the condition that $F(4) = 2$ then we began by first finding the general anti-derivative, i.e. $\frac{1}{4}x^4 + C$, then force it to be the case that if we plug in 4 for x , the output is 2. That is

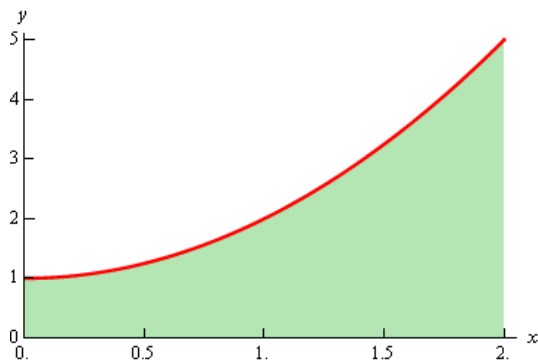
$$2 = \frac{1}{4}(4)^4 + C = 64 + C$$

and so $C = -62$ and we see that $F(x) = \frac{1}{4}x^4 - 62$.

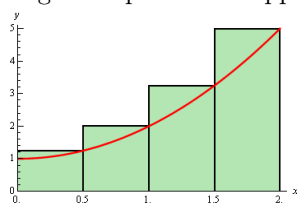
Now let's practice, find the specified anti-derivative of the following:

- (a) $10x^5 + 3x^2 + 5x$ where $F(0) = 1$
 - (b) $\frac{3}{x^2}$ where $F(3) = 1$
 - (c) $3\sin(\pi x)$ where $F(2) = 0$
3. **The Fate of The Furious Mathematician**¹ Imagine you're racing a speed car whose brakes produce a constant deceleration of 30 ft/s/s. If the car is traveling at 88 ft/s when you apply the brakes, how far will it travel before coming to a complete stop?

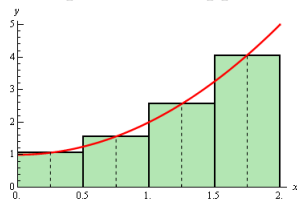
¹Do try this at home!

4. Accumulated Change²

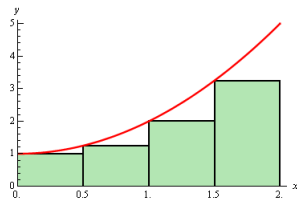
- Right endpoint area approximation: Find the area under the curve $y = x^2 + 1$ on $[0, 2]$ for $n = 4$.



- Mid-point area approximation: Find the area under the curve $y = x^2 + 1$ on $[0, 2]$ for $n = 4$.

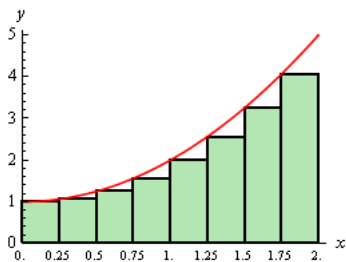


- Left endpoint area approximation: Find the area under the curve $y = x^2 + 1$ on $[0, 2]$ for $n = 4$.



5. Riemann sum

Use Riemann sums and left endpoints to find the exact area under $y = x^2 + 1$ from $x = 0$ to $x = 2$.



²All graphs from Paul's Online Notes <http://tutorial.math.lamar.edu/>