

Please show **all** your work! Answers without supporting work will not be given credit.

Name: _____

1. **Logistics**

- Office Hours: MS 3957, Tuesday at 10a and Thursdays at 11a
- SMC: MS 3974 Monday-Thursday 9a-3p (I'll be there Thursdays at 10a)

2. **Even More Extreme**

- (a) Let x be the total number of people at Coachella. The cost of organizing Coachella for x people is given by $C(x) = 0.0002x^3 - 60x^2 + 800000000000$. If the average ticket costs \$600, how many tickets should Coachella sell to maximize its profit? What is this maximum profit?
- (b) Suppose we are building an aquarium. The sides of the aquarium are made of glass, the bottom is made of metal, and the top is made of metal. The aquarium should be twice as long as it is wide and should have a total volume of 4000 cm^3 . If metal costs \$3 per square centimeter and the glass costs \$2 per square centimeter, what should the dimensions of the aquarium be to minimize its cost to build?¹
- (c) Yacoub is drowning in the ocean (call this Point A). The nearest point of shore is 50 meters away (call this Point B). You are on the shoreline, 40 meters away from Point B (call this Point C). You swim at a rate of 1 m/s and run at a rate of 2 m/s. Yacoub refuses to let you save him unless you take the time-optimal path. What should your path be? Assume the shoreline is perfectly straight and that you actually want to save Yacoub.

¹Jeremy Brightbill shared this warm up problem.

3. **Summation notation** To write the sum $c_1 + c_2 + c_3 + \dots + c_n$ in a more succinct way we use the capital Greek letter sigma, \sum . We write an indexing variable i that goes through the integers one-by-one starting from the number under the sigma and ending at the number above the sigma:

$$c_1 + c_2 + c_3 + \dots + c_n = \sum_{i=1}^n c_i. \text{ Write } 1 + 2 + \dots + 100 \text{ using sigma notation. Evaluate } \sum_{k=1}^6 k.$$

4. Linear Regression

Review:

- You have a bunch of data points $(x_1, y_1), \dots, (x_n, y_n)$. These are fixed constants, you cannot change these. Your ‘variable(s)’ will be m (and later in multivariable calculus b).
- You think there’s a straight line (that is, the graph of $y = mx + b$) that fits these points well. Not all relations are linear, but sometimes we can do transformations to make them linear.
- To make this idea of ‘fitting well’ precise, we define an error score $E = \sum_{i=1}^n (y_i - (mx_i + b))^2$. A smaller error is a better fit. There are other error scores but this is the most common one.
- Your goal is usually to find the slope m and the intercept b that minimizes the error, however, as this is univariate calculus, we will give b or say that the line passes through the origin, i.e. $b = 0$.
- We minimize per usual by first finding critical points $\frac{dE}{dm} = \sum_{i=1}^n 2(y_i - (mx_i + b))(-x_i) =$

$$\sum_{i=1}^n -2y_i x_i + mx_i^2 + bx_i = 0, \text{ which has solution}$$

$$m = \frac{\sum_{i=1}^n x_i y_i - \sum_{i=1}^n b x_i}{\sum_{i=1}^n x_i^2}.$$

Practice: Find the line of best fit through the points (1,2), (4,6), (7,11), and (10,15) and passes through the origin.