Math 3B — Week 2 Discussion Practice Problems — April 24, 2017

Please show **all** your work! Answers without supporting work will not be given credit.

Name:______________________________________

1. **Logistics**
   - Office Hours: MS 3957, Tuesday at 10a and Thursdays at 11a
   - SMC: MS 3974 Monday-Thursday 9a-3p (I'll be there Thursdays at 10a)

2. **Warm up**

   Use polynomial long division on the following:

   (a) Divide $x^2 + 2x + 1$ by $x + 1$.

   (b) Divide $x^2 + 2x + 1$ by $x - 1$.

   (c) Divide $4x^2 + 3x + 4$ by $2x + 1$.

   (d) Divide $x^3 - 3x^2 + 1$ by $x^2 + 1$.

3. **Curve Sketching**

   Review:

   - Oblique Asymptotes (Asymptotes of the form $y = ax + b$, use polynomial long division)

   Sketch the following functions. Label all asymptotes and intercepts. Label all the intervals where the function is increasing. Label all intervals where the function is concave up.

   (a) $f(x) = \dfrac{-3x^2 + 4x + 1}{x - 1}$

   (b) $f(x) = \dfrac{e^{-(\ln x)^2}}{x}$

4. **Extremization**

   Review:

   - Local Extrema: These can occur at endpoints or critical points (places where the first derivative is zero or undefined) but it must be in the domain.
   - First Derivative Test: Check the sign of the first derivative **around** the critical point.
   - Second Derivative Test: Check the sign of the second derivative **at** the critical point.
   - Global Extrema: Compare the output value at all local extrema and endpoints. In case of open interval domain, endpoint output values are computed via limits and may cause global extrema to **not** exist.

   Find any and all local and global extrema. Use **both** first and second derivative tests on each.

   (a) $f(x) = x^3 - 75x + 100$

   (b) $f(x) = x + \frac{1}{x}$ on the interval [0.1,2].

   (c) $f(x) = \sin(x)$ on the interval $(0, \pi)$.

   Suppose $f(t) = \dfrac{6}{1 + 2e^{-t}}$ models the population of a small island. Find the starting population. Show that the population as $t \to \infty$ is 6. Solve $f(t) = \frac{6}{2}$ for t; let this be t^*. Then show that the population is growing most rapidly when the population is $\frac{6}{2} = 3$. That is, show that $f'(t)$ is maximal when $t = t^*$. (Hint: here, the first derivative test requires computing $f''(x)$ and the second derivative test requires $f'''(x)$.)

The End.