Homework Assignment 1 – Math 273a, Fall 2015

- This homework assignment is open to all textbooks (listed above or not), class notes, and Internet documents except for any solution manual or the solutions from the previous quarters.
- You are encouraged to ask questions and discuss the questions and solutions on http://piazza.com/ucla/fall2015/math273a/home. However, copying others solutions or programs is considered a **serious violation**.
- Please type your answers in Latex and **submit a single PDF file**. Note that you can insert an external PDF file to your latex file by using

```
\usepackage{pdfpages} % add this line before \begin{document}

\includepdf[pages={1,2,3}]{myfile.pdf}
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- Although not required, you are encouraged to use notation similar to the course slides.
- Please **do not include your name anywhere in the submitted PDF file**.

Problem 1. (10 points)

[Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 2. (10 points)

[Start a problem on a new page. Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 3. (10 points)

[Start a problem on a new page. Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 4. (10 points)

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Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 5. (10 points)

[Start a problem on a new page. Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 6. (10 points)

Consider $\mathbb{R}^n$ and any iteration of the following form

$$x^{k+1} = x^k + \alpha_k \cdot p^k,$$

where α is a scalar, for solving the unconstrained optimization problem $\min_x f(x)$, where p^k is a descent direction (i.e., $\nabla f(x^k)^T p^k < 0$)and α_k satisfies the Armijo-Wolfe conditions

$$f(x^k + \alpha_k p^k) \le f(x^k) + c_1 \alpha_k \nabla f(x^k)^T p^k \tag{1a}$$

$$\nabla f(x^k + \alpha_k p^k)^T p^k \ge c_2 \nabla f(x^k)^T p^k, \tag{1b}$$

where $0 < c_1 < c_2 < 1$. Suppose that $f(x)$ is bounded below and that $f(x)$ is continuously differentiable and the gradient $\nabla f(x)$ is Lipschitz continuous, that is, there exists a constant $L > 0$ such that

$$\|\nabla f(x) - \nabla f(y)\| \le L\|x - y\|, \quad \forall x, y \in \mathbb{R}^n.$$

1. (5 pts.) Prove
$$\sum_{k\ge 0} \cos^2 \theta_k \|\nabla f(x^k)\|^2 < \infty,$$
where $\cos\theta_k = \frac{-\nabla f(x^k)^T p^k}{\|\nabla f(x^k)\|\|p^k\|}$.

2. (5 pts.) Show that steepest gradient method with line search which satisfies the Wolfe conditions converges to a saddle point.

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]