Fall 2016 290J: Introduction to Mean Field Games

Organizers: Wilfrid Gangbo and Inwon Kim

Time: Mondays 4:30-6pm, in MS 5138.

This is a participating seminar for graduate students, where students make presentations and discuss together. Our aim is to learn the fast-developing theory of mean field games, by studying the recent pioneering article by Cardaliaguet et al [1].

The mean-field framework was developed to study, as N grows to infinity, systems of N number of rational agents competing with optimal strategy to maximize their value function ("Nash system"). Such systems arise naturally in many applications. The systematic study of these problems was started, in the mathematical community by Lasry and Lions [3], and independently around the same time in the engineering community by Caines, Huang, and Malham [4].

Heuristically, in the large N limit one finds a " differential game" where each (infinitesimal) player optimizes his payoff, depending upon the collective behavior of the others. This formal reasoning lead to the *MFG system*, which is a deterministic coupled equation for the value function and the population density. Prior to [1] most available results concerned this system.

It turns out that the MFG system is not sufficient to prove convergence result for the Nash system. To establish the connection with N-player system, one is led to the *Master equation*, where one considers the population density as a probability measure, and study the evolution of the value function in the space of probability measures. If classical solutions of the master solution exists, it not only yields the solutions for MFG system but also provides an approximation to the Nash system. [1] serves as the first paper where one proves these results under monotonicity assumptions and with the presence of diffusion(noise) in the system. Without these assumptions most questions regarding master equation remain open: the best result at the moment is that of Gangbo and Święch [5.], who proves a short-time version of the result.

Content of the seminar: We wish to obtain a good understanding of the current status of mean field games theory over the next two quarters. In the fall, we aim to study mainly chapter 3 of article [1], which concerns the well-posedness of the first-order master equation where the setting is relatively simple. Our aim is to carefully go through together the details of arguments, including the background materials. In the winter quarter we plan to proceed further in the article, in particular to the convergence results. In the first week we will have an organizational meeting and start presentations in week two.

Prerequisites: Most arguments in [1] would require understanding of basic measure theory. Prior exposure to PDE theory and/or probability theory may be helpful, but not necessary. While the Nash system is written in terms of stochastic differential equations, we will not touch this topic at least in the fall quarter.

References:

1. Cardaliaguet, Pierre, Franois Delarue, Jean-Michel Lasry, and Pierre-Louis Lions. "The master equation and the convergence problem in mean field games." arXiv preprint arXiv:1509.02505 (2015).

2. Gomes, Diogo A. "Mean field games models - a brief survey." Dynamic Games and Applications 4, no. 2 (2014): 110-154.

3. Lasry, Jean-Michel, and Pierre-Louis Lions. "Mean field games." Japanese Journal of Mathematics 2, no. 1 (2007): 229-260.

4. Huang, Minyi, Roland P. Malhamé, and Peter E. Caines. "Large population stochastic dynamic games: closed-loop McKean-Vlasov systems and the Nash certainty equivalence principle." Communications in Information and Systems 6, no. 3 (2006): 221-252.

5. Gangbo, Wilfrid, and Andrzej Święch. "Existence of a solution to an equation arising from the theory of mean field games." Journal of Differential Equations 259, no. 11 (2015): 6573-6643.