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Erratum to "Corrections to: Weyl Sums for Quadratic Roots"

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In the paper [2], to prove Theorem 7.1, we apply our old result about sums of Salié sums (Theorem 4 of [1]) and this is then applied to prove Lemma 9.1. Although these applications go through without any slips, unfortunately [1, Theorem 4] turns out to be incorrect; specifically, the main term in (1.11) of [1] should be slightly different from the one claimed. We owe these observations to Lilu Zhao and are grateful to him for pointing them out.

Fortunately, the exact form of the main term (1.11) of [1] does not play a role in the application to this paper [2], so all the conclusions remain unchanged. Indeed, our Theorem 7.1, which comes by recalling [1, Theorem 4], does not contain a main term, however, the condition "aq not a square" must be replaced by "$aq/(aq, r^\infty)$ not a square". Moreover, in the comments following Theorem 7.1, aq must be replaced by $aq/(aq, r^\infty)$ twice. Consequently, when we move on to the application to Lemma 9.1, we must change the sentence after (9.3) to "Note that $n/(n, r^\infty)$ can be a square only if $n \mid r$". Then, six lines later, we need to change the ending of the paragraph into "say, where $T'(x)$ and $T'''(x)$ are the partial sums over $r \le R$ such that $n/(n, r^\infty)$ is, or is not, a square, respectively, and $T''(x)$ is the partial sum over $R < r \le x$". Lastly, we need to replace the estimations

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for $T'''(x)$ leading to (9.6) by the following:

$$T'''(x) \le \sum_{\substack{c\le x\\ n|c}} c^{-\frac{1}{2}} |K(n, n; c)| \le n^{\frac{1}{2}} \sum_{\substack{c\le x\\ n|c}} \tau(c),$$

which replaces (9.6) by the bound

$$T'''(x) \ll n^{-\frac{1}{2}} x \log x.$$

This bound is slightly stronger than what we had claimed. Thus Lemma 9.1 holds as stated.

We hope that the above alterations will not make it difficult to study our paper and we apologize to the reader for any inconvenience this may cause.

Our source paper [1] also requires some corrections. Fortunately, they are not substantial, so we would like to take this opportunity to present them here.

(A) The main term of the formula (1.11) of Theorem 4 of [1] should be

$$24\pi^{-2} \varepsilon_b \, \delta(a_1) \frac{\psi(ar)a}{\varphi(r)} \left(\frac{a_2}{b}\right) \operatorname{Re}(E(x/2a)).$$

(B) In the next line after (1.11) replace "... we define $\delta(a) = 1$ if a is ..." by "... we define $a_1 a_2 = a$ with $(a_1, r) = 1$, $a_2 \mid r^\infty$, $\delta(a_1) = 1$ if a_1 is ...".

(C) On page 25, line-4, the main term should be

$$12\pi^{-2} \varepsilon_b \frac{\psi(ar)a}{\varphi(r)} \{x + \cdots\}.$$

(D) In (5.7), the outer summation over $\beta \pmod{rn}$ needs the extra condition $\beta n \equiv b \pmod{r}$.

Remark. It was the accidental omission of this congruence $\beta n \equiv b \pmod{r}$ which has necessitated all of the corrections.

(E) The formula (5.10) should be

$$\sideset{}{^*}\sum_{\substack{\beta \pmod{rn}\\ \beta n \equiv b \pmod{r}}} \left(\frac{\beta}{a_2}\right) e\left(4a\frac{\bar{\beta}}{n}\right) = \mu(n) \left(\frac{bn}{a_2}\right).$$

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(F) The main term in (5.11) should be

$$2\delta(a_1)\varphi(a_1)a_2\frac{\mu(n)}{rn^2}\left(\frac{bn}{a_2}\right)\operatorname{Re}(E(x/2a)).$$

(G) In (5.12), the Legendre symbol (n/a) should be (n/a_1).

(H) The main term in (5.13) should be

$$\frac{12}{\pi^2 r}\left(\frac{b}{a_2}\right)\delta(a_1)\varphi(a_1)a_2\prod_{p|ar}(1-p^{-2})^{-1}\operatorname{Re}(E(x/2a)).$$

(I) After (5.16), we should have said “and the main term is equal to the one claimed in (1.11). This completes the proof of Theorem 4.”

References

[1] Duke, W., J. B. Friedlander, and H. Iwaniec. “Bilinear forms with Kloosterman fractions.” *Inventiones Mathematicae* 128, no. 1 (1997): 23–43.

[2] Duke, W., J. B. Friedlander, and H. Iwaniec. “Weyl sums for quadratic roots.” *IMRN* (published online, June 21, 2011).

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