Math 33B
Midterm 2

Friday, November 16, 2007

Name: _____

Student ID: _____

Signature: _____

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. (10 pts) Given that $y_1 = t^{-1}\cos t$ and $y_2 = t^{-1}\sin t$ are solutions of the differential equation

$$ty'' + 2y' + ty = 0,$$

find the general solution of the differential equation

$$ty'' + 2y' + ty = \sec t$$

2. (a) (8 pts) Find the general solution of the differential equation:

$$y'' + 4y' + 5y = 20\cos(5t)$$

(b) (2 pts) From your answer in part (a), write down the steady-state response in the form $A\cos(\omega t - \phi)$:

3. (a) (5 pts) Find the solution of the system of equations

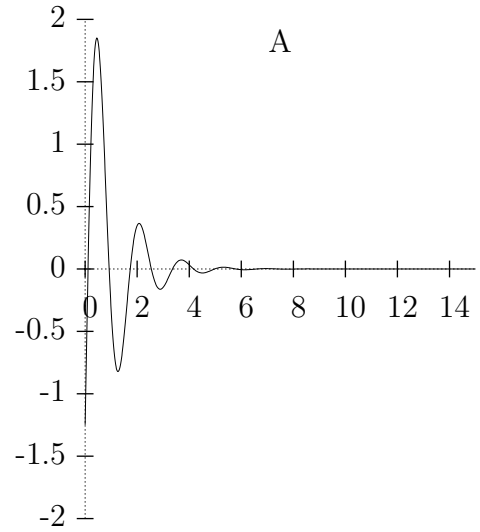
$$\mathbf{y}' = \begin{pmatrix} 8 & 10 \\ -5 & -7 \end{pmatrix} \mathbf{y}, \qquad \mathbf{y}(0) = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

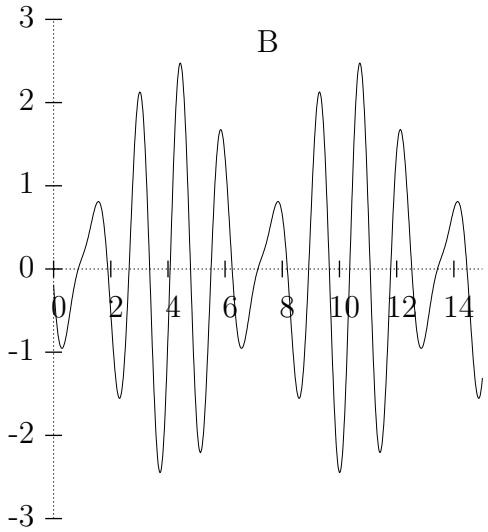
(b) (5 pts) Find the solution of the system of equations

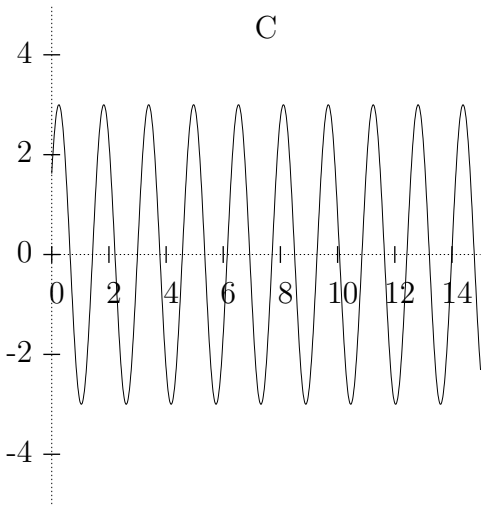
$$\mathbf{y}' = \begin{pmatrix} 0 & 1 \\ -4 & -4 \end{pmatrix} \mathbf{y}, \qquad \mathbf{y}(0) = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

4. (10 pts) Match the following five differential equations with the graphs on the next page, and describe the type of harmonic motion as one of the following: "Undamped", "Underdamped", "Critically damped", "Overdamped", "Interference", or "Resonance".

Differential Equation	Graph	Description
$y'' + 16y = 12\cos(5t)$	_____	_____
$y'' + 16y = 0$	_____	_____
$y'' + 8y' + 16y = 0$	_____	_____
$y'' + 16y = 12\cos(4t)$	_____	_____
$y'' + 2y' + 16y = 0$	_____	_____


A
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 2 4 6 8 10 12 14


B
3
2
1
0
-1
-2
-3
0 2 4 6 8 10 12 14


C
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0
-2
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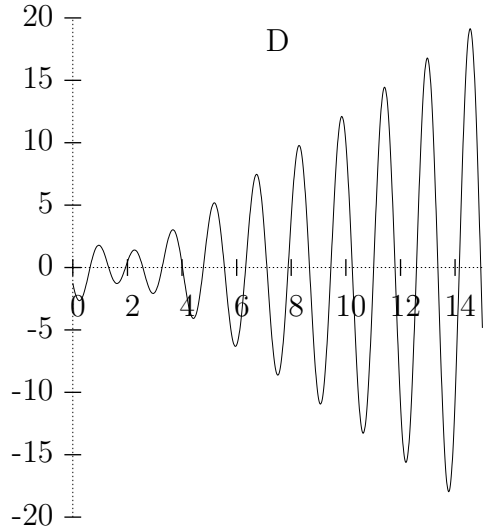
0 2 4 6 8 10 12 14


D
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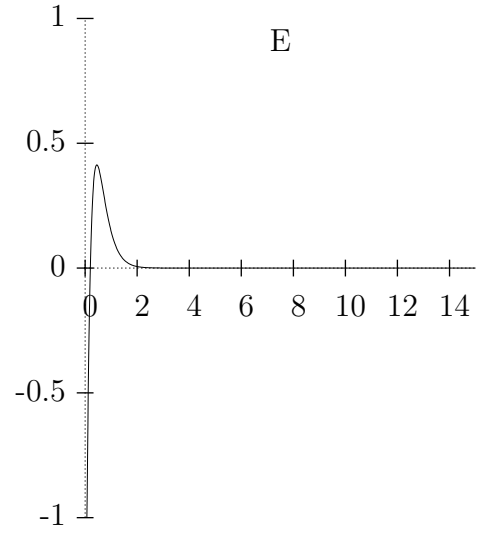
15
10
5
0
-5
-10
-15
-20
0 2 4 6 8 10 12 14


E
1
0.5
0
-0.5
-1
0 2 4 6 8 10 12 14