Math 33B Midterm 1

Wednesday, October 24, 2007

Name: _____

Student ID:

Signature:

Problem	Max	Score
1	8	
2	10	
3	12	
4	10	
Total	40	

1. Find the general solution of the differential equation

$$(x^{2} + y^{2})dx + (x^{3} + 2xy + 3xy^{2})dy = 0$$

Hint: Look for an integrating factor depending only on y.

2. Consider the differential equation

$$t^2y'' + ty' - y = 0$$

(a) Show that y = t is a solution.

(b) Find the general solution.

3. The Green Canyon Lake is fed by the Rushing River. Due to a dam upstream, the flow rate of the river is given by the function

$$f(t) = 2 + \cos(t) \quad \frac{\mathrm{km}^3}{\mathrm{day}}$$

Water flows out of the lake via a spillway at the same rate, to keep its volume constant at 20 km³. A factory on the river causes the water entering the lake to have a constant concentration of 10 kg of pollutant per km³. Assume that at t = 0, there is no pollutant in the lake.

(a) Set up an initial value problem to model the amount y of pollutant in the reservoir as a function of time t (measured in days).

(b) Using qualitative analysis, determine the quantity of pollutant in the lake in the long run.

(c) Solve the initial value problem from part (a) explicitly to find the quantity of pollutant in the lake at any time t.

4. Find the general solution of each of the following differential equations: (a) y'' - 10y' + 25y = 0

(b)
$$y'' + 4y' + 29y = 0$$

(c)
$$y'' - 3y' - 10y = 0$$