Math 33B Homework 9

1. (a) Compute e^{At} for $A = \begin{pmatrix} 5 & 3 \\ -6 & -4 \end{pmatrix}$.

   (b) Use this to solve $\mathbf{y}' = A\mathbf{y}, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

   (c) Is the equilibrium at the origin a sink, source, saddle, center, spiral sink/source, or degenerate sink/source?

2. (a) Compute e^{At} for $A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$.

   (b) Use this to solve $\mathbf{y}' = A\mathbf{y}, \quad \mathbf{y}(-1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (c) Is the equilibrium at the origin a sink, source, saddle, center, spiral sink/source, or degenerate sink/source?

3. (a) Compute e^{At} for $A = \begin{pmatrix} 6 & -3 \\ 15 & -6 \end{pmatrix}$.

   (b) Use this to solve $\mathbf{y}' = A\mathbf{y}, \quad \mathbf{y}(1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

   (c) Is the equilibrium at the origin a sink, source, saddle, center, spiral sink/source, or degenerate sink/source?

4. (a) Compute e^{At} for $A = \begin{pmatrix} -1 & 1 \\ -4 & -5 \end{pmatrix}$.

   (b) Use this to solve $\mathbf{y}' = A\mathbf{y}, \quad \mathbf{y}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

   (c) Is the equilibrium at the origin a sink, source, saddle, center, spiral sink/source, or degenerate sink/source?

5. Compute e^{At} for the matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ -2 & 4 & -1 \end{pmatrix}$.

   Hint: To find the eigenvalues, take the determinant by expanding along the second row, or use a computer.

6. Compute e^{At} for the matrix $A = \begin{pmatrix} -4 & 0 & 3 \\ 3 & -1 & -3 \\ -6 & 0 & 5 \end{pmatrix}$.

   Hint: To find the eigenvalues, take the determinant by expanding along the second column, or use a computer.

For the next four problems, the given $n \times n$ matrix A has just one eigenvalue of algebraic multiplicity n. Find the eigenvalue, then find a complete list of n "generalized eigenvectors" (state the "level" of each). Finally, write down the general solution of the system of equations $\mathbf{y}' = A\mathbf{y}$.

Note: You do not need to compute e^{At}, but if you do these problems in the most straightforward way, you could compute it easily!

Hint: If a matrix is *upper-triangular* (i.e. all of its entries below the diagonal are 0) or *lower-triangular* (i.e. all of its entries above the diagonal are 0) then its determinant is the product of the entries on the diagonal.

7. $A = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{pmatrix}$

8. $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

9. $A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

10. $A = \begin{pmatrix} -5 & 1 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & -5 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix}$