Math 115A
Midterm 2

Monday, November 15, 2010

Name: ____________________

Student ID: ____________________

Signature: ____________________

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. (10 pts) Let $T : V \to W$ and $S : W \to Z$ be linear transformations. Prove that $ST = T_0$ if and only if $R(T) \subseteq N(S)$. (Recall that T_0 denotes the "zero map": $T_0(x) = 0$ for all $x \in V$.)

2. (10 pts) Let V and W be n-dimensional vector spaces, and let $T : V \to W$ be a linear transformation. Let $\{v_1, \ldots, v_n\}$ be a basis for V. Prove that T is an isomorphism if and only if $\{T(v_1), \ldots, T(v_n)\}$ is a basis for W.

3. (a) (5 pts) Write down a formula for a linear map $T : P_2(\mathbb{R}) \to \mathbb{R}^3$ such that

$$\begin{aligned} T(X^2) &= (3, 1, -2), \\ T(X^2 + X) &= (1, -2, 1), \text{ and} \\ T(X^2 + X + 1) &= (-3, 6, -3). \end{aligned}$$

(Your answer should be in the form $T(a + bX + cX^2) = \dots$.)

(b) (5 pts) Compute rank(T) and nullity(T). Is T an isomorphism?

4. Define $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ by $T(f) = f(1) \cdot X^2 + f'$. Let $\beta = \{X^2 - X - 3, X^2 + 2X + 1, 3X - 2\}$ and let $\gamma = \{X^2, X, 1\}$.

   (a) (4 pts) Compute the matrix $[T]_\gamma$.

   (b) (4 pts) Let Q be the change of coordinate matrix that changes β-coordinates to γ-coordinates. Compute Q.

   (c) (2 pts) Using your answers from parts (a) and (b), write down an expression for the matrix $[T]_\beta$. (You do not need to multiply out the matrices.)