Math 115A
Midterm 1

Monday, October 18, 2010

Name: ______________________________

Student ID: ______________________________

Signature: ______________________________

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. Let $V = P_2(\mathbb{Q})$, the vector space of polynomials of degree at most 2 with rational coefficients, and let $\beta = \{1 - X + X^2, 1 + X^2, X - X^2\}$.

   (a) (6 pts) Prove that β is a basis for V.

   (b) (4 pts) Write $3 - 2X + 5X^2$ as a linear combination of elements of β.

2. (10 pts) Let $V = M_{2\times 2}(\mathbb{R})$, the vector space of all 2×2 matrices with real entries. Let

$$W_1 = \left\{ \begin{pmatrix} a & b \\ b & d \end{pmatrix} \in V \;\middle|\; a, b, d \in \mathbb{R} \right\},$$
$$W_2 = \left\{ \begin{pmatrix} 0 & -c \\ c & 0 \end{pmatrix} \in V \;\middle|\; c \in \mathbb{R} \right\}.$$

Then W_1 and W_2 are subspaces of V. (You don't need to prove this.) Prove that $V = W_1 \oplus W_2$.

3. (10 pts) Let V be a vector space, and let W_1 and W_2 be subspaces of V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if either $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

4. (10 pts) Let V be a vector space over $\mathbb{C}$ of dimension n. Recall (from a homework exercise) that V is also then automatically a vector space over $\mathbb{R}$. Prove that, as a vector space over $\mathbb{R}$, the dimension of V is $2n$. *(Hint: Start with a basis for V as a vector space over $\mathbb{C}$, and modify it somehow to get a basis for V as a vector space over $\mathbb{R}$.)*