Math 115A
Midterm

Tuesday, August 24, 2010

Name: ______________________________

Student ID: ______________________________

Signature: ______________________________

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. Let V be a vector space over a field F.

   (a) Let $a \in F$ and $v, w \in V$. Prove that if $a \neq 0$ and $av = aw$ then $v = w$.

   (b) Let $a, b \in F$ and $v \in V$. Prove that if $v \neq 0$ and $av = bv$ then $a = b$.

2. Define $T : \mathbb{R}^3 \to P_2(\mathbb{R})$ by

$$T(a, b, c) = (a + c)X^2 + (a + b)X + (a + 3b - 2c).$$

(a) Find a basis for $R(T)$.

(b) What is $\operatorname{rank}(T)$? Is T onto?

(c) What is $\operatorname{nullity}(T)$? Is T one-to-one?

3. Recall that a function $f : \mathbb{R} \to \mathbb{R}$ is called *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$, and is called *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$.

   (a) Let $f : \mathbb{R} \to \mathbb{R}$ be any function. Define functions f_1 and f_2 by

   $$f_1(x) = \frac{f(x) - f(-x)}{2} \qquad \text{and} \qquad f_2(x) = \frac{f(x) + f(-x)}{2}.$$

   Prove that f_1 is odd and f_2 is even.

   (b) Let V be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$, and let W_1 be the subspace of all odd functions and W_2 be the subspace of all even functions. Prove that $V = W_1 \oplus W_2$. *(Hint: Use part (a)!)*

4. Let $V = P_2(\mathbb{R})$ and $W = P_3(\mathbb{R})$, and define a linear transformation $T : V \to W$ by

$$T(f) = \int_1^X f(t)dt.$$

Let $\beta = \{6X^2 - 2X, 4X + 1, 2X - 3\}$ and $\gamma = \{1, X + 1, X^2 + 1, X^3 + 1\}$ (so β and γ are ordered bases of V and W, respectively.) Compute $[T]_\beta^\gamma$.