## Math 110A Midterm 2

Monday, July 19, 2010

Name: \_\_\_\_\_

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Student ID:

Signature:

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

- 1. Let R be a ring. Recall that an element a of R is called *nilpotent* if there exists a positive integer n such that  $a^n = 0_R$ .
  - (a) Explain what it would mean to say that the property of being a nilpotent element of R is "preserved by homomorphisms".

(b) Prove the statement you made in part (a). (In other words, prove that being a nilpotent element of R is preserved by homomorphisms.)

2. Let R be an integral domain, and let  $a, b \in R$ . Show that a and b are associates if and only if  $a \mid b$  and  $b \mid a$ .

- 3. Let  $p = X^4 3X^3 4X^2 + 19X 7$  and  $q = X^3 4X^2 + 2X + 7$  in  $\mathbb{Q}[X]$ .
  - (a) Use the Euclidean algorithm to compute the greatest common divisor of p and q. (You may want to use the back of this page for scratch work.)

(b) Using your answer from part (a), show that p is reducible in  $\mathbb{Q}[X]$  but does not have any roots in  $\mathbb{Q}$ .

4. Factor the polynomial  $X^3 + 3X^2 + 3X + 4$  into irreducibles in  $\mathbb{Z}_5[X]$ . Show your work, and justify your answer. (In particular, be sure to explain briefly why each factor is irreducible.)