Math 110A
Midterm 1

Tuesday, July 6, 2010

Name: ______________________________

Student ID: ______________________________

Signature: ______________________________

Problem	Max	Score
1	15	
2	15	
3	10	
4	10	
Total	50	

1. Solve the following congruences. If there are no solutions, say so, and give some justification. (You do not need to give a complete proof.) If there are multiple solutions, be sure to find all of them.

   (a) $15x + 10 = 4$ in $\mathbb{Z}_{20}$

   (b) $15x + 10 = 4$ in $\mathbb{Z}_{19}$

   (c) $15x + 10 = 4$ in $\mathbb{Z}_{18}$

2. (a) Prove that $10^n \equiv 1 \pmod 9$ for all $n \geq 0$. *(Hint: Induction)*

(b) Let x be a positive integer, and let y be the sum of the digits of x (in base 10). Prove that $x \equiv y \pmod 9$. *(Hint: Think about how to write x in terms of its digits. Use part (a).)*

(c) Use part (b) to prove that a positive integer is divisible by 9 if and only if the sums of its digits is divisible by 9.

3. Let R be a ring, and let $a \in R$. Let

$$S = \{ax \mid x \in R\},$$
$$T = \{xa \mid x \in R\}.$$

(a) Show that for all $r \in R$ and $s \in S$, $sr \in S$.

(b) Show that for all $r \in R$ and $t \in T$, $rt \in T$.

(c) Show that S and T are subrings of R.

4. (a) Let R be a commutative ring with identity, and let $a, b \in R$ such that ab is a unit. Show that a and b are units.

(b) Now let R be a nonzero ring with identity, and assume R has no zero divisors. (Note: R is *not* necessarily commutative.) Let $a, b \in R$ such that ab is a unit. Show that a and b are units.