Inquiry in Numbers

"I hear, I forget. I see, I remember. I do, I understand." - Confucius

Description of the course:

This is a course designed to teach non-mathematicians to love mathematics and to see math as a mathematician does: not as a means to an end, but as beautiful and artful in its own right. The subject of the course is elementary number theory, the study of whole numbers. From a humble beginning, requiring as a prerequisite only a firm grasp of high school algebra, we will develop a rich and elegant theory of prime numbers, factorization, and modular arithmetic.

What is novel about this course is not the subject matter, but the unique method of teaching and learning. The course will be taught in the inquiry-based learning method. Unlike a standard math class, where the professor gives lectures and writes proofs on the blackboard, this class is entirely student-driven. The instructors prepare "scripts" that consist of a list of statements that, step by step, lead to major theorems to be discussed in the class. It's up to the students to understand and prove each statement at home. Students are encouraged not to look at textbooks or other outside sources, and instead come up with solutions on their own, or in collaboration with classmates. During the class period, the instructors select volunteers to present solutions, and it's up to the rest of the class to listen critically to determine whether the solution is correct – and if it isn't, to find the correct one together. In particular, the instructors will not announce when a student completes a proof or makes an error. They will simply moderate the discussion. At the end of a script, each student will prepare a journal of carefully written solutions to each of the problems on the script. These journals, along with class participation, will be the main sources of evaluation for the course.

This style of teaching puts the responsibility of learning on the student. It fosters creativity by allowing students to choose their own path toward a solution, rather than simply following along with a textbook or a lecture. It also creates a dialog in the class: professors are taken at their word ("the professor never makes a mistake!"), but when peers present a proposed solution, the other students have to listen intently and ask questions. Moreover, because everyone has worked on all the problems before coming to class, each student can actively compare the presented solution with they own, and can catch aspects of it that either they or the presenter missed.

Students taught this way end up with a deeper understanding of the material. They also learn a number of transferable skills along the way, including communication and presentation skills from lecturing to the class, group work from collaborating with peers, and writing skills from preparing journals.

This style of teaching is very demanding for the instructor, which is why we propose that the course be taught by a team of two. At any given time, one instructor is moderating the discussion, while the other is evaluating the students' presentation and participation.

Outline of the course:

The course work will be broken up into 7 scripts. At the start of each new script, an instructor will give a brief (10-30 minutes) lecture introducing the material and providing some motivating examples. When necessary, the instructor will also give provide a summary at the end of the script. The rest of the class time will be devoted to student presentations. A week-by-week summary of the material to be covered:

- Week 1. Warm-up – properties of the natural numbers and their subsets
- Week 2. Divisibility in the integers
- Week 3. The division algorithm
- Week 4. Greatest common divisor and its properties, Euclidean algorithm
- Week 5. Primes, fundamental theorem of arithmetic
- Week 6. Infinitude of primes, growth of number of primes
- Week 7. Congruence in the integers
- Week 8. Fermat's little theorem, Chinese remainder theorem
- Week 9. Primality testing
- Week 10. Which primes may be written as a sum of two squares?

This covers the most important points in classical number theory. If the students are more advanced or particularly motivated, additional topics can be covered, such as quadratic reciprocity, quadratic forms, or an introduction to Diophantine equations.

Proposed texts:

The main course for the class will be the scripts prepared by the instructors. Students are discouraged from consulting outside sources.

Evaluation:

The students' grade will be based on class participation, journals, and a written final exam. The breakdown will be:

- Class participation: 40%
- Journals: 40%
- Exam: 20%

Class participation: Class participation means presenting a proof, making an insightful comment, answering a question, or otherwise adding to the discussion. At the end of each class period, each instructor will record a participation grade of 0, 1, or 2 for each student. A student can earn a grade of 2 by making several significant contributions, or many small contributions; a grade of 1 means the student participated but not very significantly (for example, did not present anything but asked two questions); and a grade of 0 means the student added little or nothing to the class discussion.

Journals: Each journal will correspond to exactly one script. Hence, the journals will be due about a week after the material of each script has been completed, in order to give the students time to write the solutions clearly and well. The journals will be graded not just on the correctness of the solutions, but also the quality and clarity of the writing. Indeed, students should use this as an opportunity to develop their writing skills.

Final exam: The written final exam will be in-class and closed-book. About a week before the final exam, the students will be given a collection of problems, a subset of which will comprise exam. This way, the students can prepare their solutions ahead of time, just like they have been doing all quarter, and then “present” them to the instructor in writing. Thus, the exam tests the students’ understanding of the material as well as their writing skills.

Disciplines and majors:

The course has no prerequisites and is designed for the motivated and intellectually curious student in any discipline or major. This could include anyone: from the arts student who had aptitude for math in high school but was put off by college calculus, to the engineering student who has extensive experience with applying math to the real world and now wants to see the beauty in numbers themselves.

Proposed number of units:

This course carries a standard workload and so we propose the typical 5 units.

Enrollment:

The ideal number of students for the course would be 10-20. This number allows the instructors to get to know the students personally, and tailor the course to their strengths and interests. It also gives each student a chance to participate in each lecture, and for the instructors to accurately gauge their level of participation.

Preferred quarter:

Since the course has no prerequisites, no quarter is preferred over another.

Meeting schedule:

Ideally the course should meet for two 80 minute periods per week, on Tuesday and Thursday. The four-day break between Thursday and Tuesday gives the students sufficient time to prepare their solutions for the next week. The 80 minute period gives enough time so that each student can present at each meeting. However, this is not strictly necessary and a schedule of three 50 minute meetings per week would also be workable.

TA support:

As we propose the course enrollment be small, a TA will not be needed.

Division:

Although the course has no prerequisites, it does require a great deal of self-motivation and academic maturity. For this reason, we suggest the course be upper division.