HOMEWORK 6

- Section 4.5 in the book: Exercises 18, 20, 22, 26, 38, 42, 44.

Problem 1. Consider the equation

$$(2-t)x''' + (2t-3)x'' - tx' + x = 0 \quad \text{for} \quad t < 2.$$

(a) Verify that $\phi_1(t) = e^t$ is a solution.
(b) Look for a solution of the form $\phi_2(t) = v(t)\phi_1(t)$. Plug this into the equation and derive a differential equation for v. Solve it.
(c) Write down the general solution to the equation.

Problem 2. Consider the equation

$$t^2y'' + 7ty' + 5y = t \quad \text{for} \quad t > 0.$$

(a) Verify that $\phi_1(t) = t^{-1}$ is a solution to the associated homogeneous problem.
(b) Look for a solution to the inhomogeneous problem of the form $Y(t) = v(t)\phi_1(t)$. Plug this into the equation and prove that v' must satisfy a first order linear differential equation. Solve it and find Y.

Problem 3. Consider the equation

$$(1-2t-t^2)x'' + 2(1+t)x' - 2x = 0.$$

(a) Verify that $\phi_1(t) = t+1$ is a solution to the equation.
(b) Let ϕ_2 be a second solution to the differential equation so that $W(\phi_1, \phi_2)(0) = 1$. Use Abel's theorem to find the Wronskian determinant of ϕ_1 and ϕ_2 at all times $t \in (-1-\sqrt{2}, -1+\sqrt{2})$.
(c) Use part (b) to find all possible solutions ϕ_2 satisfying $W(\phi_1, \phi_2)(0) = 1$.