HOMEWORK 3

- Section 2.6 in the book: Exercises 10, 12, 20, 26, 28, 30.

Problem 1. Show that the following differential equation is exact

$$y + x\frac{dy}{dx} = 0.$$

Find an *explicit* formula for the solution $y(x)$. What is the interval of existence of the solution?

Problem 2. This problem is based on Problem 50 in Section 2.6 in the book; in particular, see Figure 9.
a) Show that $\tan(\theta) = \tan(2\beta)$ and use this to derive the *correct* form of equation (6.45). (Note that the formula in the book is *not* correct.)
b) Use part (a) and the quadratic formula to derive an equation for y'. Show that this equation for y' can be rearranged to give

$$\Big(\frac{x}{\sqrt{x^2+y^2}} - 1\Big)\,dx + \frac{y}{\sqrt{x^2+y^2}}\,dy = 0. \tag{1}$$

c) Show that the differential equation (1) is exact on the rectangle $R = (0,\infty) \times (0,\infty)$ and solve it.