

HOMEWORK 6

- Section 4.5 in the book: Exercises 18, 20, 22, 26, 38, 42, 44.

Problem 1. Find the general solutions to the following differential equations:

(a) $x^{(4)} + 2x'' + x = 0$,

(b) $x^{(5)} + 5x^{(4)} - 2x''' - 10x'' + x' + 5x = 0$.

Problem 2. Consider the equation

$$(2 - t)x''' + (2t - 3)x'' - tx' + x = 0 \quad \text{for } t < 2.$$

- (a) Verify that $\phi_1(t) = e^t$ is a solution.
(b) Look for a solution of the form $\phi_2(t) = v(t)\phi_1(t)$. Plug this into the equation and derive a differential equation for v . Solve it.
(c) Write down the general solution to the equation.

Problem 3. Consider the equation

$$t^2y'' + 7ty' + 5y = t \quad \text{for } t > 0.$$

- (a) Verify that $\phi_1(t) = t^{-1}$ is a solution to the associated homogeneous problem.
(b) Look for a solution to the inhomogeneous problem of the form $Y(t) = v(t)\phi_1(t)$. Plug this into the equation and prove that v' must satisfy a first order linear differential equation. Solve it and find Y .

Problem 4. Consider the equation

$$(1 - 2t - t^2)x'' + 2(1 + t)x' - 2x = 0.$$

- (a) Verify that $\phi_1(t) = t + 1$ is a solution to the equation.
(b) Let ϕ_2 be a second solution to the differential equation so that $W(\phi_1, \phi_2)(0) = 1$. Use Abel's theorem to find the Wronskian determinant of ϕ_1 and ϕ_2 at all times $t \in (-1 - \sqrt{2}, -1 + \sqrt{2})$.
(c) Use part (b) to find all possible solutions ϕ_2 satisfying $W(\phi_1, \phi_2)(0) = 1$.