33B Visan Midterm 2 February 20th

First Name: _____ ID#_____

Last Name: _____

Section: _____ $= \begin{cases} 1a & \text{Tuesday with S. Kim} \\ 1b & \text{Thursday with S. Kim} \\ 1c & \text{Tuesday with J. Murphy} \\ 1d & \text{Thursday with J. Murphy} \\ 1e & \text{Tuesday with F. Robinson} \\ 1f & \text{Thursday with F. Robinson} \end{cases}$

Rules:

- There are **FOUR** problems for a total of 40 points.
- Use the backs of the pages.
- No calculators, computers, notes, books, e.t.c..
- Out of consideration for your classmates, no chewing, humming, pen-twirling, snoring, e.t.c.. Try to sit still.
- Turn off your cell-phone.

1	2	3	4	$\sum$

(1) (10 points)

Find the general solution to the equation

$$x'' - 2x' - 3x = 3te^{2t}.$$

(2) (10 points)

Consider the equation

$$t^2x'' - tx' + x = 4t\ln(t) \quad \text{for} \quad t > 0.$$

(a) Verify that $\phi_1(t) = t$ and $\phi_2(t) = t\ln(t)$ form a fundamental set of solutions to the corresponding homogeneous equation for $t \in (0, \infty)$.

(b) Find a particular solution to the given inhomogeneous equation.

(c) Write down the general solution to the equation.

(3) (10 points)

Find the solution to the following initial-value problem

$$x'' - 8x' + 17x = 0 \quad \text{with} \quad x(0) = 4 \quad \text{and} \quad x'(0) = -1.$$

(4) (10 points)

Consider the equation

$$(3t+1)x'' - (9t+6)x' + 9x = 1 \quad \text{for} \quad t > -\tfrac{1}{3}. \tag{1}$$

(a) Verify that $\phi_1(t) = e^{3t}$ is a solution to the associated homogeneous equation.

(b) Look for a solution to equation (1) of the form $\phi_2(t) = v(t)\phi_1(t)$. Plug this into the equation and derive a differential equation for $v(t)$. Find the general solution for $v(t)$.

(c) Write down the general solution to equation (1).

(d) Write down a fundamental set of solutions to the associated homogenous equation

$$(3t+1)x'' - (9t+6)x' + 9x = 0 \quad \text{for} \quad t \in (-\tfrac{1}{3}, \infty).$$