HOMEWORK 5

• Section 4.5 in the book: Exercises 22, 38, 42.

Problem 1. Find the general solutions to the following differential equations:

$$(a) \quad x^{(4)} + 2x'' + x = 0,$$

$$(b) \quad x^{(5)} + 5x^{(4)} - 2x''' - 10x'' + x' + 5x = 0.$$

Problem 2. Consider the equation

$$(2-t)x''' + (2t-3)x'' - tx' + x = 0 \quad \text{for} \quad t < 2.$$

(a) Verify that $\phi_1(t) = e^t$ is a solution.
(b) Look for a solution of the form $\phi_2(t) = v(t)\phi_1(t)$. Plug this into the equation and derive a differential equation for v. Solve it.
(c) Write down the general solution to the equation.

Problem 3. Find the general solution to the equation

$$4x'' - 4x' + x = e^{t/2}\sqrt{1-t^2}.$$

Problem 4. Use the method of undetermined coefficients (or the method of annihilators) to find the general solution to the equation

$$x'' + x = t\cos(t) - \cos(t).$$

Problem 5. Consider the equation

$$tx'' - (1+t)x' + x = t^2e^{2t} \quad \text{for} \quad t > 0.$$

(a) Verify that $\phi_1(t) = 1 + t$ and $\phi_2(t) = e^t$ form a fundamental set of solutions to the corresponding homogeneous equation for $t \in (0, \infty)$.

(b) Find a particular solution to the given inhomogeneous equation.

(c) Write down the general solution to the inhomogeneous equation.