

HOMework 3

- Section 2.6 in the book: Exercises 10, 12, 26, 28, 30.
- Section 2.7 in the book: Exercises 8, 10, 30, 32.
- Section 2.9 in the book: Exercises 10, 18, 22.

Problem 1. Consider the differential equation

$$\frac{dx}{dt} = |x|^{1/2}(1 - x).$$

- Explain why for any $x_0 \in (-\infty, \infty)$ there exists a solution to the equation satisfying the initial condition $x(0) = x_0$, at least on some time interval containing 0.
- Give an example of a rectangle in the tx plane on which the existence and uniqueness (Picard) theorem does not apply. Justify your answer.
- Identify the equilibrium points.
- Draw a phase diagram and identify the stable and unstable points.
- Sketch the equilibrium solutions in the tx plane. These equilibrium solutions divide the tx plane into regions. Sketch at least one solution curve in each of these regions.
- For the particular solution with initial condition $x(0) = 0.43$, what is the limit $\lim_{t \rightarrow \infty} x(t)$?

Problem 2. Determine whether the Picard Theorem guarantees that the differential equation

$$\frac{dx}{dt} = \sqrt{x^2 - 9}$$

admits a unique solutions through the following points:

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|----------------------|-----------------------|
| <i>a</i>) $(-1, 1)$ | <i>c</i>) $(5, 3)$ |
| <i>b</i>) $(2, -3)$ | <i>d</i>) $(1, 4)$. |