First Name: ____________________ ID#____________________

Last Name: ____________________

Section: ____________________ $= \begin{cases} 1a & \text{Tuesday with M. Bulkow} \\ 1b & \text{Thursday with M. Bulkow} \\ 1c & \text{Tuesday with L. Vera} \\ 1d & \text{Thursday with L. Vera} \\ 1e & \text{Tuesday with A. Mennen} \\ 1f & \text{Thursday with A. Mennen} \end{cases}$

Rules.

- There are **FIVE** problems; ten points per problem.
- This is a 50 minute exam.
- Use the backs of the pages.
- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.
- Turn off your cell-phone, pager,...

1	2	3	4	5	$\sum$

(1) Indicate TRUE/FALSE or fill in the missing number, as appropriate:

T / F : The intersection of two subspaces is a subspace.

T / F : $\dim\bigl(\operatorname{span}\{\vec{v}_1, \vec{v}_2\}\bigr) = 2$ for any $\vec{v}_1$, $\vec{v}_2$ in $\mathbb{R}^5$.

If A is a 3×5 matrix then $\dim(\operatorname{im}(A)) + \dim(\ker(A)) =$ ______.

T / F : Reversing the order of the rows in a 4×4 matrix does not change the determinant.

If A is an invertible 5×5 matrix then $\dim(\operatorname{im}(A)) \times \dim(\ker(A)) =$ ______.

T / F : If $A\vec{x} = A\vec{y}$ then $x - y \in \ker(A)$.

If $\vec{x} \cdot (A\vec{y}) = 0$ for all $\vec{x}$ and $\vec{y}$ then $\dim(\operatorname{im}(A)) =$ ______.

T / F : If R is the matrix of a rotation through $180°$, then $\det(R) = -1$.

If $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ then $\dim(\operatorname{im}(A)) =$ _____, and $\dim(\ker(A)) =$ _____.

(2) Consider two bases for $\mathbb{R}^4$. First, the standard basis $\mathcal{S} = (\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4)$ and secondly $\mathcal{B} = (\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$ where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad \vec{v}_4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 4 \end{bmatrix}$$

(a) If $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, find $[\vec{x}]_{\mathcal{S}}$.

(b) If the linear transformation T has $[T]_{\mathcal{B}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$ find $[T]_{\mathcal{S}}$

extra paper

(3) (a)Finish this definition:

We say that a subset W of $\mathbb{R}^m$ is a (*linear*) *subspace* if and only if ...

(b) Let P denote the plane passing through the points $\begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$.

(Notice that this plane does not pass through the origin.)

Find the point on P that is nearest to $\begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$.

(4) (a)Finish this definition:

A square matrix Q is called *orthogonal* if and only if ...

(b) Find an orthonormal basis for $\mathrm{im}(A)$ where $A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 3 & 2 \\ 1 & 4 & 1 \end{bmatrix}$.

(c) Express the third column of A in terms of this orthonormal basis.

(5) (a)Finish this definition:

We say that vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ are *linearly independent* if and only if ...

(b) Compute the determinant of

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \\ 4 & 4 & 3 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{extra paper}}$