HOMEWORK 5

Due on Monday, May 4th, in class.

Exercise 1. (20 points) In each of the following cases, find the solution to the given initial-value problem.

$$y'' - 2y' - 3y = 3te^{2t}, \quad y(0) = 1, \quad y'(0) = 0$$
$$y'' + 2y' + 5y = 4e^{-t}\cos(2t), \quad y(0) = 1, \quad y'(0) = 0.$$

Exercise 2. (10 points) Consider the equation

$$y'' - 3y' - 4y = 2e^{-t}.$$

- Find a fundamental set of solutions to the associated homogeneous equation.
- Look for a solution to the nonhomogeneous problem of the form $Y(t) = v(t)e^{-t}$. Plug this into the equation and prove that v must satisfy the differential equation $v'' - 5v' = 2$. Solve it and conclude that

$$Y(t) = -\frac{2}{5}te^{-t} + c_1e^{4t} + c_2e^{-t}, \quad \text{for some } c_1, c_2 \in \mathbb{R}.$$

Exercise 3. (20 points) In each of the following cases, verify that the given functions ϕ_1, ϕ_2 are solutions to the corresponding homogeneous equation. Then find a particular solution to the given nonhomogeneous equation.

$$ty'' - (1+t)y' + y = t^2e^{2t}, \quad t > 0; \quad \phi_1(t) = 1 + t, \quad \phi_2(t) = e^t$$
$$(1-t)y'' + ty' - y = 2(t-1)^2e^{-t}, \quad 0 < t < 1; \quad \phi_1(t) = e^t, \quad \phi_2(t) = t.$$

Exercise 4. (10 points) Consider the equation

$$t^2y'' + 7ty' + 5y = t, \quad t > 0.$$

- Verify that $\phi_1(t) = t^{-1}$ is a solution to the associated homogeneous problem.
- Look for a solution to the nonhomogeneous problem of the form $Y(t) = v(t)\phi_1(t)$. Plug this into the equation and prove that v' must satisfy a first order linear differential equation. Solve it and find Y.

Exercise 5. (20 points) In each of the following cases, determine the general solution to the given nonhomogeneous equation.

$$y''' - y'' - y' + y = 2e^{-t} + 3$$
$$y^{(4)} + y''' = \sin(2t).$$

Exercise 6. (10 points) Given that t, t^2, and t^{-1} are solutions to the homogeneous equation corresponding to

$$t^3y''' + t^2y'' - 2ty' + 2y = 2t^4, \quad t > 0,$$

determine a particular solution to the given nonhomogeneous problem.

Exercise 7. (10 points) Use the method of variation of parameters to find the general solution to

$$y^{(4)} + 2y'' + y = \sin t.$$