HOMEWORK 4

Due on Monday, April 27th, in class.

Exercise 1. (10 points) Determine the intervals in which the solutions are sure to exist in each of the following cases:

$$ty''' + (\sin t)y'' + 3y = \cos t$$
$$t(t-1)y^{(4)} + e^t y'' + 4t^2 y = 0.$$

Exercise 2. (10 points) In each of the following cases determine whether the given set of functions is linearly dependent or independent. If they are linearly dependent, find a linear relation among them.

$$f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t$$
$$f_1(t) = 2t - 3, \quad f_2(t) = 2t^2 + 1, \quad f_3(t) = 3t^2 + t.$$

Exercise 3. (20 points) Consider the equation

$$(2-t)y''' + (2t-3)y'' - ty' + y = 0, \quad t < 2.$$

• Verify that $\phi_1(t) = e^t$ is a solution.
• Look for a solution of the form $\phi_2(t) = v(t)e^t$. Plug this into the equation and derive a differential equation for v. Solve it.

Exercise 4. (40 points) In each of the following cases, find the general solution to the given differential equation.

$$2y''' - 4y'' - 2y' + 4y = 0$$
$$y^{(6)} - 3y^{(4)} + 3y'' - y = 0$$
$$y^{(6)} - y'' = 0$$
$$y^{(4)} - 8y' = 0.$$

Exercise 5. (20 points) Show that for the differential equation

$$y'' + y = 0,$$

(a) there are infinitely many solutions obeying $y(0) = y(\pi) = 0$;
(b) there is exactly one solution obeying $y'(0) = 0$ and $y(\pi) = 1$; and
(c) there are no solutions obeying $y(0) = 1$ and $y(\pi) = 1$.