HOMEWORK 7

Due on Wednesday, February 25th, in class.

Exercise 1 (10 points)**.** Prove that the only subsets of the real numbers that are both open and closed are $\emptyset$ and $\mathbb{R}$.

Exercise 2 (10 points)**.** Let $a \in \mathbb{R}$. Show that the set $\{a\}$ is closed. Conclude that any finite set is closed.

Exercise 3 (10 points)**.** Let $F \subseteq \mathbb{R}$ be a closed set bounded above. Prove that the least upper bound of F belongs to F.

Exercise 4 (10 points)**.** Prove that a set F is closed if and only if every convergent sequence in F has the property that its limit belongs to F.

Exercise 5 (50 points)**.** For each of the following sets decide if they are open, closed, or not open and not closed, compact or not compact. Also, in each case write down the set of accumulation points. Justify your answers.

1) $A = \mathbb{Q}$.
2) $A = \mathbb{Q} \cap [0, 1]$.
3) $A = \{(-1)^n(1 + \frac{1}{n})\}$.
4) $A = \bigcup_{n\in\mathbb{N}}[n, n + \frac{1}{n}]$.
5) $A = \bigcup_{n\in\mathbb{N}}[\frac{1}{2^{n+1}}, \frac{1}{2^n}]$.