131A Visan Practice Midterm May 7th

First Name: _____ ID#_____

Last Name: _____

Rules:

- There are **FOUR** problems for a total of 100 points.
- Use the backs of the pages.
- No calculators, computers, notes, books, e.t.c..
- Out of consideration for your classmates, no chewing, humming, pen-twirling, snoring, e.t.c.. Try to sit still.
- Turn off your cell-phone.

1	2	3	4	$\sum$

Problem 1. Let $x \in \mathbb{R}$ be an irrational number. Prove that $7x+3$ is not a rational number.

Problem 2. Let $a \geq 0$ be a real number. Prove by mathematical induction that

$$(1+a)^n \geq 1 + na + \frac{n(n-1)}{2}a^2 \quad \text{for all} \quad n \in \mathbb{N}.$$

Problem 3. Let $\{a_n\}_{n\geq 1}$ be a sequence defined by the following rule:

$$a_1 = 3 \quad \text{and} \quad a_{n+1} = \frac{a_n}{2} + \frac{3}{2a_n} \quad \text{for all} \quad n \geq 1.$$

- (10 points) Assuming that $\{a_n\}_{n\geq 1}$ converges to $l > 0$, find l. Justify your steps!
- (10 points) Prove that l found above is a lower bound for the sequence $\{a_n\}_{n\geq 1}$.
- (10 points) Show that $\{a_n\}_{n\geq 1}$ is monotonically decreasing.
- (5 points) Deduce that $\{a_n\}_{n\geq 1}$ converges.

Problem 4. Decide whether the following series diverges or converges. *Justify* your answer.

$$\sum_{n\geq 2} \frac{1}{(\ln n)^{\ln n}}.$$

Scratch Paper