

## HOMEWORK 2

**Exercise 1.** (10 points) Show that  $\sqrt{15}$  and  $(2 + \sqrt{2})^{1/2}$  do not represent rational numbers.

**Exercise 2.** Let  $a, b, c$  be real numbers. Show that

$$2ab \leq a^2 + b^2$$

and

$$ab + bc + ca \leq a^2 + b^2 + c^2.$$

Specify what axioms you are using at each step.

**Exercise 3.** Let  $(F, +, \cdot)$  be a field with exactly four distinct elements  $F = \{0, 1, a, b\}$  where 0 and 1 denote the identities for  $+$  and  $\cdot$ , respectively, and  $a, b$  denote the remaining two elements of  $F$ . Fill in the addition and multiplication tables below. Use the axioms to justify your solution. (Note that for each table entry there is a *unique* correct solution.)

+	0	1	a	b
0				
1				
a				
b				

·	0	1	a	b
0				
1				
a				
b				

*Hint:* Show that in the addition table each row and each column contains every element of  $F$  exactly once (as in Sudoku). Show that the same is true for the rows and columns of the multiplication table that are not identically zero.

**Exercise 4.** Solve exercises 3.5 and 3.6 from the textbook.

**Exercise 5.** Solve exercises 4.7, 4.8, 4.14, and 4.16 from the textbook.