HOMEWORK 9

Due on Wednesday, November 30th, in class.

Exercise 1. Solve exercises 28.4, 28.6, 28.8, 28.10, and 28.15 from the textbook.

Exercise 2. Solve exercises 29.2, 29.3, 29.5, 29.7, 29.12, 29.14, 29.16, and 29.18 from the textbook.

Exercise 3. Let $f : [a, b] \to \mathbb{R}$ be a continuous function on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Show that for any $c \in (a, b)$ that is not a point of maximum or minimum for f' there exist $x_1, x_2 \in (a, b)$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Exercise 4. Let $f : [a, b] \to \mathbb{R}$ be a continuous function on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Assume that f' is strictly increasing. Show that for any $c \in (a, b)$ such that $f'(c) = 0$ there exist $x_1, x_2 \in [a, b]$, $x_1 < c < x_2$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Exercise 5. Let $f : (a, b) \to \mathbb{R}$ be differentiable and let $c \in (a, b)$. Suppose that $\lim_{x \to c} f'(x)$ exists and is finite. Show this limit must be $f'(c)$.

Exercise 6. Let $f, g : [a, b] \to \mathbb{R}$ be two functions continuous on $[a, b]$ and differentiable on (a, b). Then there exists $x \in (a, b)$ such that

$$f'(x)\big[g(b) - g(a)\big] = g'(x)\big[f(b) - f(a)\big].$$

Exercise 7. Solve exercises 32.3, 32.6, 32.7, and 32.8 from the textbook.