

HOMEWORK 2

Due on Friday, October 7th, in class.

Exercise 1. (10 points) Show that $\sqrt{15}$ and $(2 + \sqrt{2})^{1/2}$ do not represent rational numbers.

Exercise 2. (20 points) Let $(F, +, \cdot, \leq)$ be an ordered field with at least two elements. Let 0 and 1 denote the identities for addition and multiplication, respectively. Prove the following statements:

1) $0 \leq a^2 \quad \forall a \in F$

2) $0 \leq 1$ and $0 \neq 1$

3) For $a \in F \setminus \{0\}$ we have $0 \leq a$ if and only if $0 \leq a^{-1}$.

4) For $a, b \in F \setminus \{0\}$ with $0 \leq a$ and $0 \leq b$ we have $a \leq b$ if and only if $a^2 \leq b^2$.

Specify what axioms you are using at each step.

Exercise 3. (20 points) Solve exercises 3.5 and 3.6 from the textbook.

Exercise 4. (10 points) Let a, b, c be real numbers. Show that

$$2ab \leq a^2 + b^2$$

and

$$ab + bc + ca \leq a^2 + b^2 + c^2.$$

Specify what axioms you are using at each step.

Exercise 5. (40 points) Solve exercises 4.7, 4.8, 4.14, and 4.16 from the textbook.