

HOMEWORK 7

Due on Friday, November 13th, in class.

Exercise 1. (15 points) Solve exercises 18.4, 18.5a), and 18.9 from the textbook.

Exercise 2. (25 points) Solve exercises 19.1, 19.3, 19.4, 19.5, and 19.7a) from the textbook.

Exercise 3. (30 points) Solve exercises 20.12, 20.13, 20.14, 20.16, 20.17, and 20.20 from the textbook.

Exercise 4. (10 points) Let $a, b \in \mathbb{R}$ with $a < b$ and let $f : [a, b] \rightarrow [a, b]$ be continuous. Show that there exists $x_0 \in [a, b]$ such that $f(x_0) = x_0$.

Exercise 5. (10 points) Let $a, b \in \mathbb{R}$ with $a < b$ and let $f, g : [a, b] \rightarrow [a, b]$ be two continuous functions such that $f \circ g = g \circ f$. Show that there exists $x_0 \in [a, b]$ such that $f(x_0) = g(x_0)$.

Exercise 6. (10 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f(x) = \begin{cases} \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

- Show that f is continuous on $\mathbb{R} \setminus \{0\}$ and discontinuous at $x = 0$.
- Show that f has the intermediate value property on \mathbb{R} .