131AH Visan Midterm November 7th

First Name: _____ ID#_____

Last Name: _____

Rules:

- There are **FOUR** problems.
- Use the backs of the pages.
- No calculators, computers, notes, books, e.t.c..
- Out of consideration for your classmates, no chewing, humming, pen-twirling, snoring, e.t.c.. Try to sit still.
- Turn off your cell-phone.

1	2	3	4	$\sum$

Problem 1. Let $(F, +, \cdot)$ be a field.

(a) List the properties that an order relation $<$ must satisfy so that $(F, +, \cdot, <)$ is an ordered field.

(b) Prove that if $(F, +, \cdot, <)$ is an ordered field, then F must contain infinitely many elements.

Problem 2. Let $\{a_n\}_{n\geq 1}$ be a bounded sequence of positive real numbers such that $\liminf_{n\to\infty} a_n > 0$. Prove that

$$\limsup_{n\to\infty} \frac{1}{a_n} = \frac{1}{\liminf_{n\to\infty} a_n}.$$

Problem 3. Let $\{x_n\}_{n\geq 1}$ be the sequence defined by the following rule:

$$x_1 = 1, \quad x_2 = 1, \quad \text{and} \quad x_{n+1} = \sqrt{x_n} + \sqrt{x_{n-1}} \quad \text{for all} \quad n \geq 2.$$

(a) Assuming that $\{x_n\}_{n\geq 1}$ converges to some real number $l \geq 0$, find l.

(b) Using induction, prove that l found above is an upper bound for the sequence $\{x_n\}_{n\geq 1}$.

(c) Show that $\{x_n\}_{n\geq 1}$ is monotonically increasing.

(d) Deduce that $\{x_n\}_{n\geq 1}$ converges.

Problem 4. Prove that the series

$$\sum_{n\geq 1} 2^{-\sqrt{n}}$$

converges.

Scratch Paper