

HOMEWORK 7

Exercise 1. If the set A has n elements and the set B has m elements, show that there are m^n many functions from A to B .

Exercise 2. For $x, y \in \mathbb{R}$ we write $x \sim y$ if $x - y$ is an integer.

a) Show that \sim is an equivalence relation on \mathbb{R} .

b) Show that the set $[0, 1) = \{x \in \mathbb{R} : 0 \leq x < 1\}$ is a set of representatives for the set of equivalence classes. More precisely, show that the map Φ sending $x \in [0, 1)$ to the equivalence class $C(x)$ is a bijection.

Exercise 3. Prove that the function $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ given by

$$f(n, m) = \frac{(n + m - 2)(n + m - 1)}{2} + n$$

is a bijection.

Hint: Use induction to show surjectivity.

Exercise 4. If A is a non-empty finite set and B is a proper subset of A , then A and B do not have the same cardinality.

Exercise 5. Show that if A is an infinite set, then A contains a countable subset.