HOMEWORK 10

Exercise 1. Let (X, d) be a metric space and let $A \subseteq X$ be complete. Show that A is closed.

Exercise 2. Let (X, d) be a complete metric space and let $F \subseteq X$ be a closed set. Show that F is complete.

Exercise 3. Consider the metric space $(X, d) = (\mathbb{R}, |\cdot|)$. For each of the following subsets of $\mathbb{R}$ decide if they are open, closed, or not open and not closed, connected or not connected. Also, in each case write down the set of accumulation points. Justify your answers.
1) $A = \mathbb{Q}$.
2) $A = \mathbb{Q} \cap [0, 1]$.
3) $A = \{(-1)^n(1 + \frac{1}{n})\}$.
4) $A = \bigcup_{n\in\mathbb{N}}[n, n + \frac{1}{n}]$.
5) $A = \bigcup_{n\in\mathbb{N}}[\frac{1}{2^{n+1}}, \frac{1}{2^n}]$.

Exercise 4. Give an example of a set $\emptyset \neq A \subsetneq \mathbb{Q}$ that that is both open and closed in $\mathbb{Q}$. Justify your answer.

Exercise 5. Assume that the sets A and B are separated and that the sets A and C are separated. Prove that the sets A and $B \cup C$ are separated.

Exercise 6. Let (X, d) be a connected metric space and let A be a connected subset of X. Assume that the complement of A is the union of two separated sets B and C. Prove that $A \cup B$ and $A \cup C$ are connected. Prove also that if A is closed, then so are $A \cup B$ and $A \cup C$.

Exercise 7. Let (X, d) be a metric space and let A, B be two closed subsets of X such that $A \cup B$ and $A \cap B$ are connected. Prove that A is connected.

Exercise 8. Let (X, d) be a complete metric space and let $\{G_n\}_{n\geq 1}$ be a sequence of dense open subsets of X. Show that $\cap_{n\geq 1} G_n$ has the Baire property.

Exercise 9. Show that $\mathbb{R} \setminus \mathbb{Q}$ has the Baire property.