HOMEWORK 7

Exercise 1. If the set A has n elements and the set B has m elements, show that there are m^n many functions from A to B.

Exercise 2. For $x, y \in \mathbb{R}$ we write $x \sim y$ if $x - y$ is an integer.

a) Show that $\sim$ is an equivalence relation on $\mathbb{R}$.

b) Show that the set $[0,1) = \{x \in \mathbb{R} : 0 \leq x < 1\}$ is a set of representatives for the set of equivalence classes. More precisely, show that the map Φ sending $x \in [0,1)$ to the equivalence class $C(x)$ is a bijection.

Exercise 3. Fix $n \geq 1$. Show that if $A_1, A_2, \ldots, A_n$ are countable sets, then the cartesian product $A_1 \times A_2 \times \ldots \times A_n$ is countable.

Exercise 4. If the sets A and B are equipotent ($A \sim B$), show that $\mathcal{P}(A) \sim \mathcal{P}(B)$.

Exercise 5. Prove that $\mathcal{P}(\mathbb{N})$ is equipotent with the set of functions

$$2^{\mathbb{N}} = \{f : \mathbb{N} \to \{0,1\} : f \text{ is a function}\}.$$

In particular, the cardinality of $\mathcal{P}(\mathbb{N})$ is $2^{\aleph_0}$.

Exercise 6. Show that $\mathbb{N}^{\mathbb{N}} \sim 2^{\mathbb{N}}$, that is, the set of sequences with values in $\mathbb{N}$ is equipotent with the set of sequences with values in $\{0,1\}$.

Exercise 7. Show that the cardinality of $\mathbb{R}$ is $2^{\aleph_0}$. You may use the fact that the interval $(0,1)$ has cardinality $2^{\aleph_0}$.

Exercise 8. Prove that the set of irrational numbers has the cardinality of $\mathbb{R}$.