

## HOMEWORK 7

**Exercise 1.** If the set  $A$  has  $n$  elements and the set  $B$  has  $m$  elements, show that there are  $m^n$  many functions from  $A$  to  $B$ .

**Exercise 2.** Fix  $n \geq 1$ . Show that if  $A_1, A_2, \dots, A_n$  are countable, then  $A_1 \times A_2 \times \dots \times A_n$  is countable.

**Exercise 3.** If  $A \sim B$ , show that  $\mathcal{P}(A) \sim \mathcal{P}(B)$ .

**Exercise 4.** Prove that  $\mathcal{P}(\mathbb{N})$  is equivalent with the set of functions

$$2^{\mathbb{N}} = \{f : \mathbb{N} \rightarrow \{0, 1\} : f \text{ is a function}\}.$$

In particular, the cardinality of  $\mathcal{P}(\mathbb{N})$  is  $2^{\aleph_0}$ .

**Exercise 5.** Show that  $\mathbb{N}^{\mathbb{N}} \sim 2^{\mathbb{N}}$ , that is, the set of sequences with values in  $\mathbb{N}$  is equivalent with the set of sequences with values in  $\{0, 1\}$ .

**Exercise 6.** Show that the cardinality of  $\mathbb{R}$  is  $2^{\aleph_0}$ . You may use the fact that the interval  $(0, 1)$  has cardinality  $2^{\aleph_0}$ .

**Exercise 7.** Prove that the set of irrational numbers has the cardinality of  $\mathbb{R}$ .