

Math 33B: 7/21

Consider one parameter family of linear systems

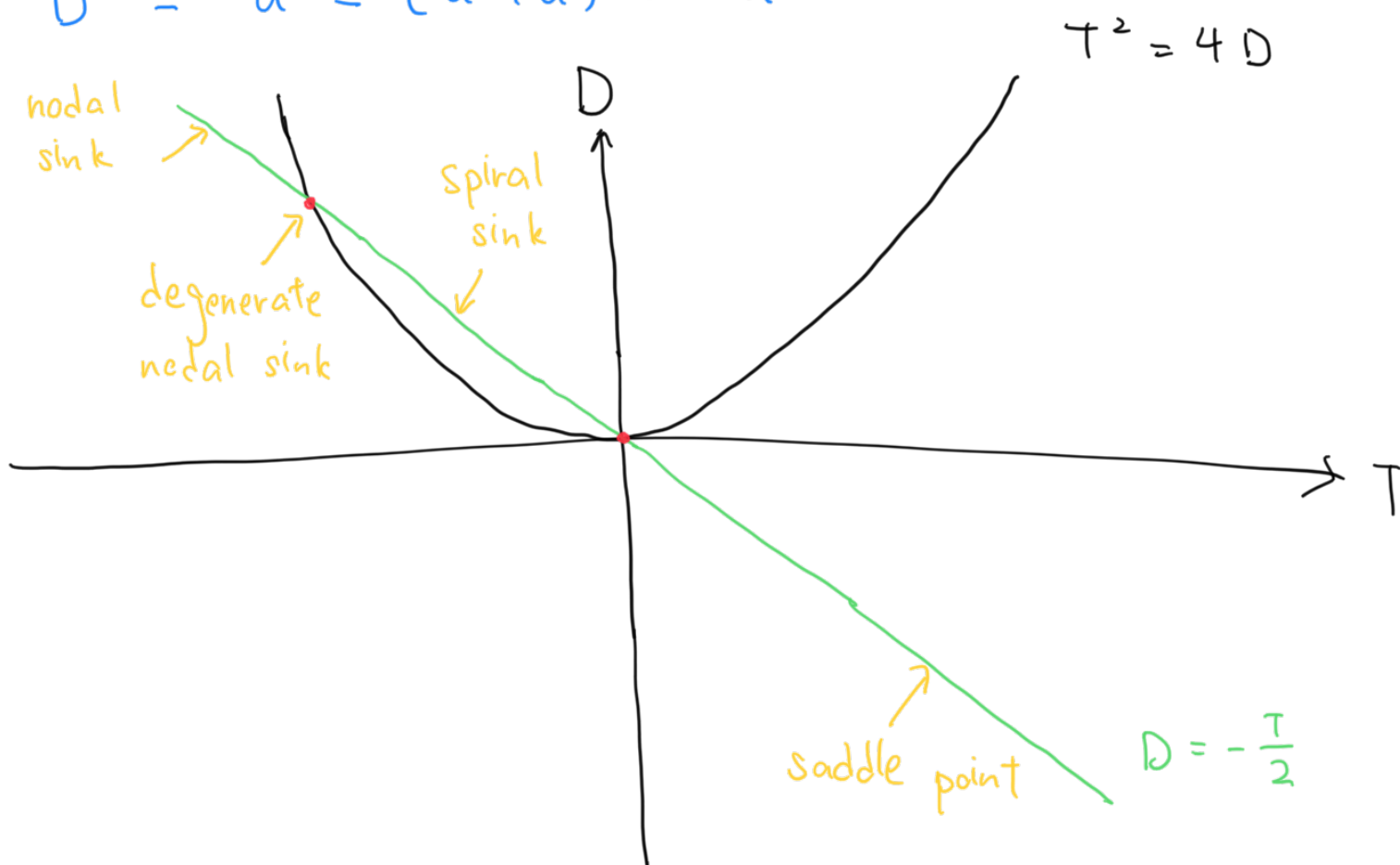
$$Y' = \begin{pmatrix} a & a^2 + a \\ 1 & a \end{pmatrix} Y$$

$$p(\lambda) = \lambda^2 - T\lambda + D$$

$$\lambda = \frac{T \pm \sqrt{T^2 - 4D}}{2}$$

$$T = a + a = 2a$$

$$D = a^2 - (a^2 + a) = -a$$



$$\begin{cases} T = 2a \\ D = -a \end{cases}$$

$\Leftrightarrow$

$$D = -\frac{T}{2}$$

$$T^2 = 4D$$

$$T^2 = 4 \cdot \left(-\frac{T}{2}\right)$$

$$T^2 + 2T = 0$$

$$T = 0 \text{ or } T = -2$$

$$T^2 = 4D$$

$$(2a)^2 = 4(-a)$$

$$a^2 + a = 0$$

$$a = 0 \text{ or } a = -1$$

$$\frac{T=0}{D=0}$$

$$\frac{T=-2}{D=1}$$

$$a = -1, \quad A = \begin{pmatrix} a & a^2+a \\ 1 & a \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$Y' = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} Y$$

$$p(\lambda) = \lambda^2 + 2\lambda + 1 = (\lambda + 1)^2$$

$$\lambda = -1$$

$$\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = - \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -a \\ a-b \end{pmatrix} = \begin{pmatrix} -a \\ -b \end{pmatrix}$$

$$a = 0$$

$$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(A - \lambda I) w = v$$

$$\left[\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

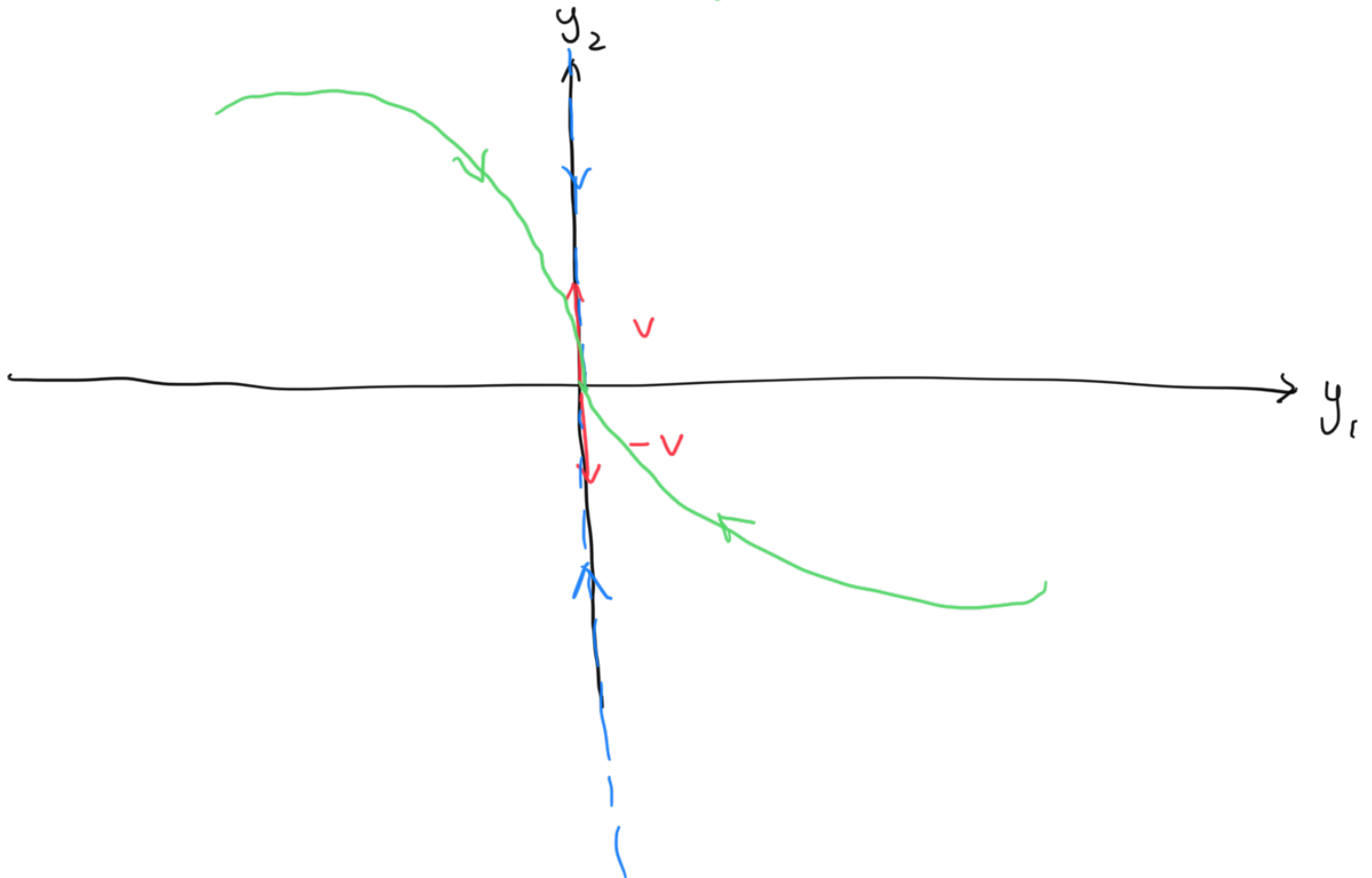
$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$C = 1$$

$$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$y(t) = C_1 \underline{e^{-t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} + C_2 e^{-t} \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$



$$a = 0$$

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$p(\lambda) = \lambda^2$$

$$\lambda = 0$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ a \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

$$a = 0$$

$$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(A - \lambda I) w = v$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$c = 1$$

$$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

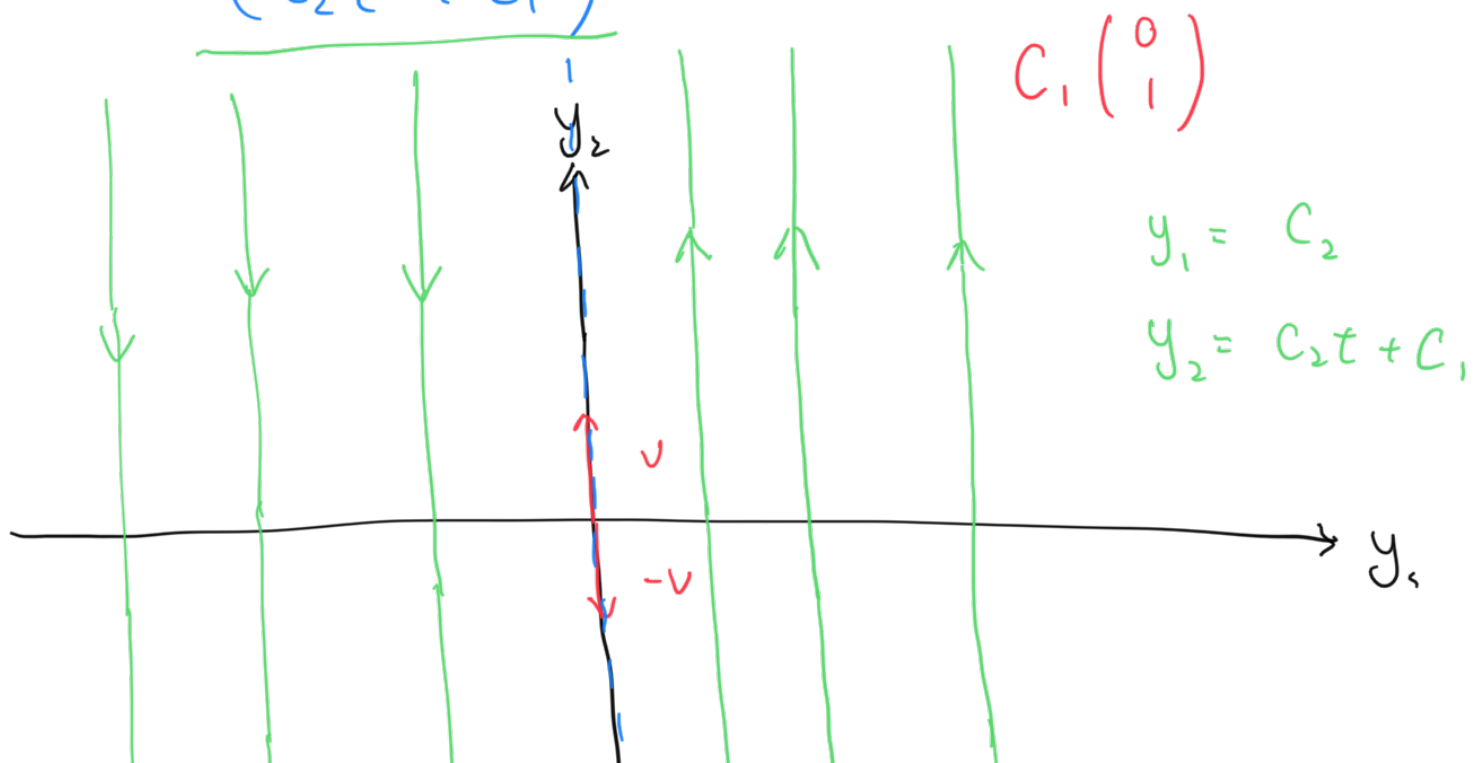
$$y(t) = \underline{C_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}} + C_2 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \underline{t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

$$Y' = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} Y$$

$$= \begin{pmatrix} 0 \\ C_1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ t \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$= \underline{\begin{pmatrix} C_2 \\ C_2 t + C_1 \end{pmatrix}}$$



B

$$T = 0$$

$$D = 0$$

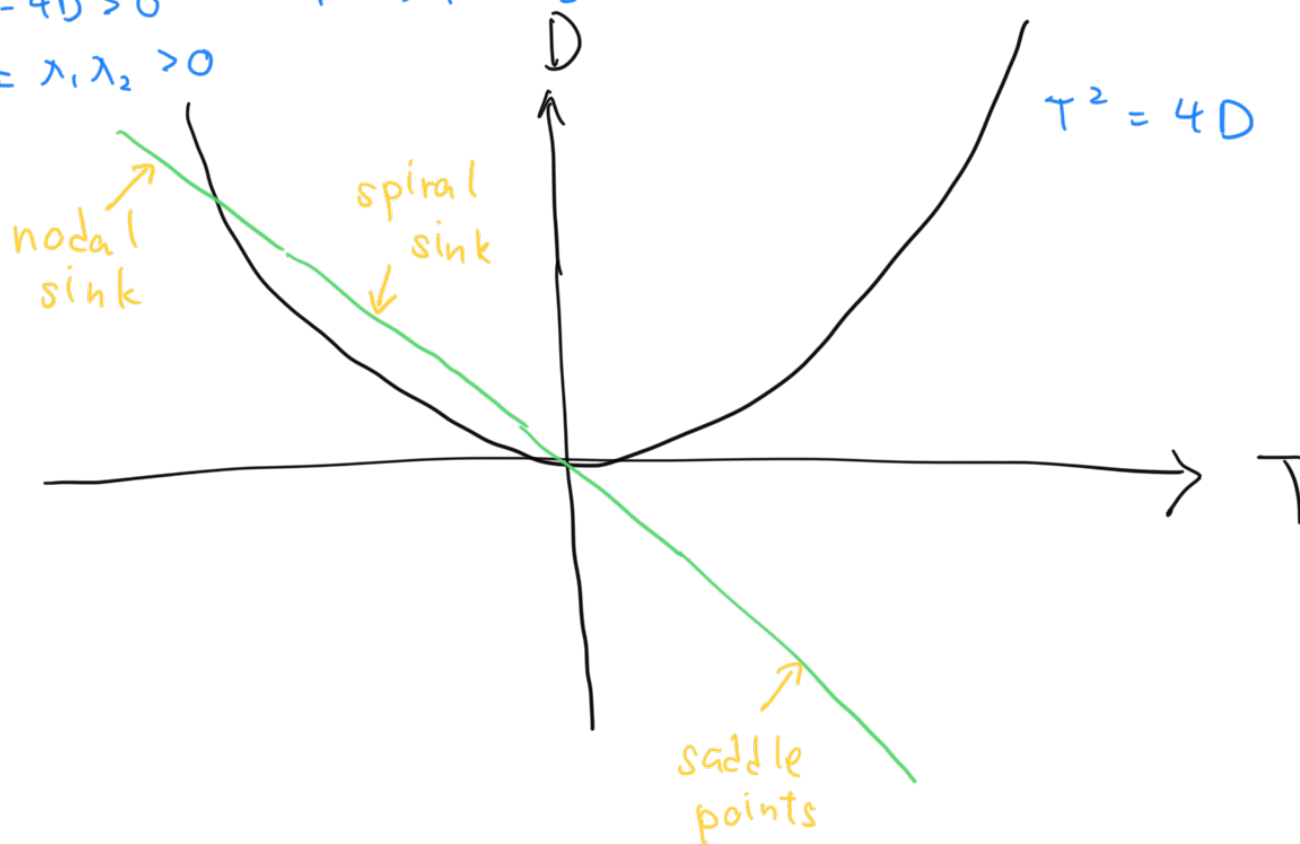
$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$T^2 - 4D > 0$$

$$D = \lambda_1 \lambda_2 > 0$$

$$T = \lambda_1 + \lambda_2 < 0$$



$$T^2 - 4D > 0$$

$$\lambda_1 < 0 < \lambda_2$$

$$D = \lambda_1 \lambda_2 < 0$$

$$T^2 - 4D < 0$$

2 complex solutions

$$\lambda = \frac{T \pm \sqrt{T^2 - 4D}}{2}$$