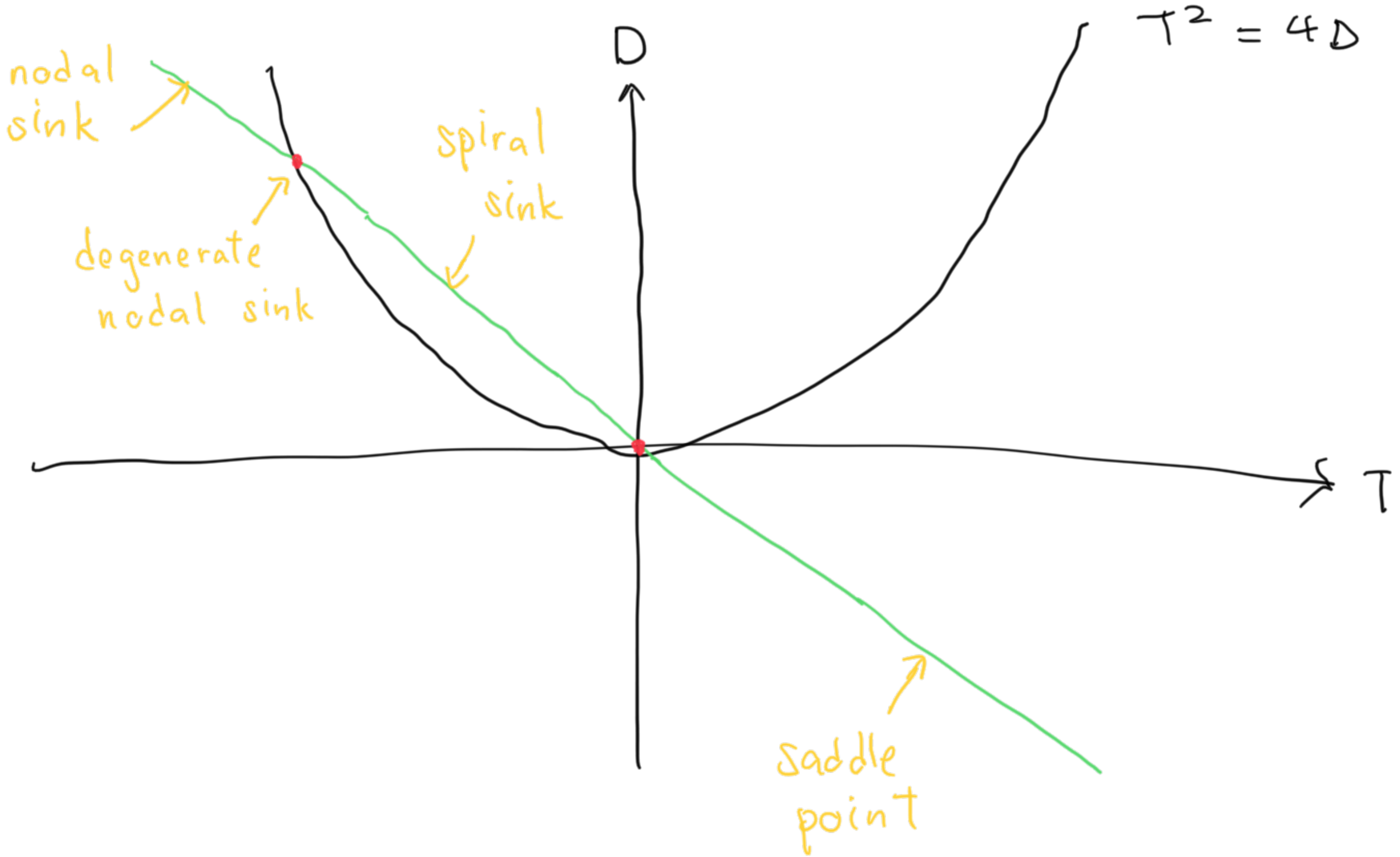
Math 33B: 7/21

$$Y' = \begin{pmatrix} a & a^2+a \\ 1 & a \end{pmatrix} Y$$

$$p(\lambda) = \lambda^2 - T\lambda + D \qquad \lambda = \frac{T \pm \sqrt{T^2-4D}}{2}$$

$$T = a + a = 2a \qquad \Longleftrightarrow \qquad D = -\frac{T}{2}$$

$$D = a \cdot a - (a^2+a) = -a$$



$$\begin{cases} T = 2a \\ D = -a \end{cases} \qquad \Longleftrightarrow \qquad D = -a = -\frac{T}{2}$$

$$a = 0$$

$$T^2 = 4D \qquad (2a)^2 = 4 \cdot (-a)$$

$$a^2 + a = 0$$

$$a = 0 \quad \text{or} \quad a = -1$$

$$T = 0 \qquad\qquad T = -2$$

$D = 0$ $\qquad\qquad$ $D = 1$

$$a = -1 \quad , \quad A = \begin{pmatrix} a & a^2 + a \\ 1 & a \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$Y' = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} Y$$

$$p(\lambda) = \lambda^2 + 2\lambda + 1 = (\lambda + 1)^2$$

$$\lambda = -1$$

$$\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -a \\ a - b \end{pmatrix} = \begin{pmatrix} -a \\ -b \end{pmatrix}$$

$$a = 0$$

$$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(A - \lambda I) w = v$$

$$\left[\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

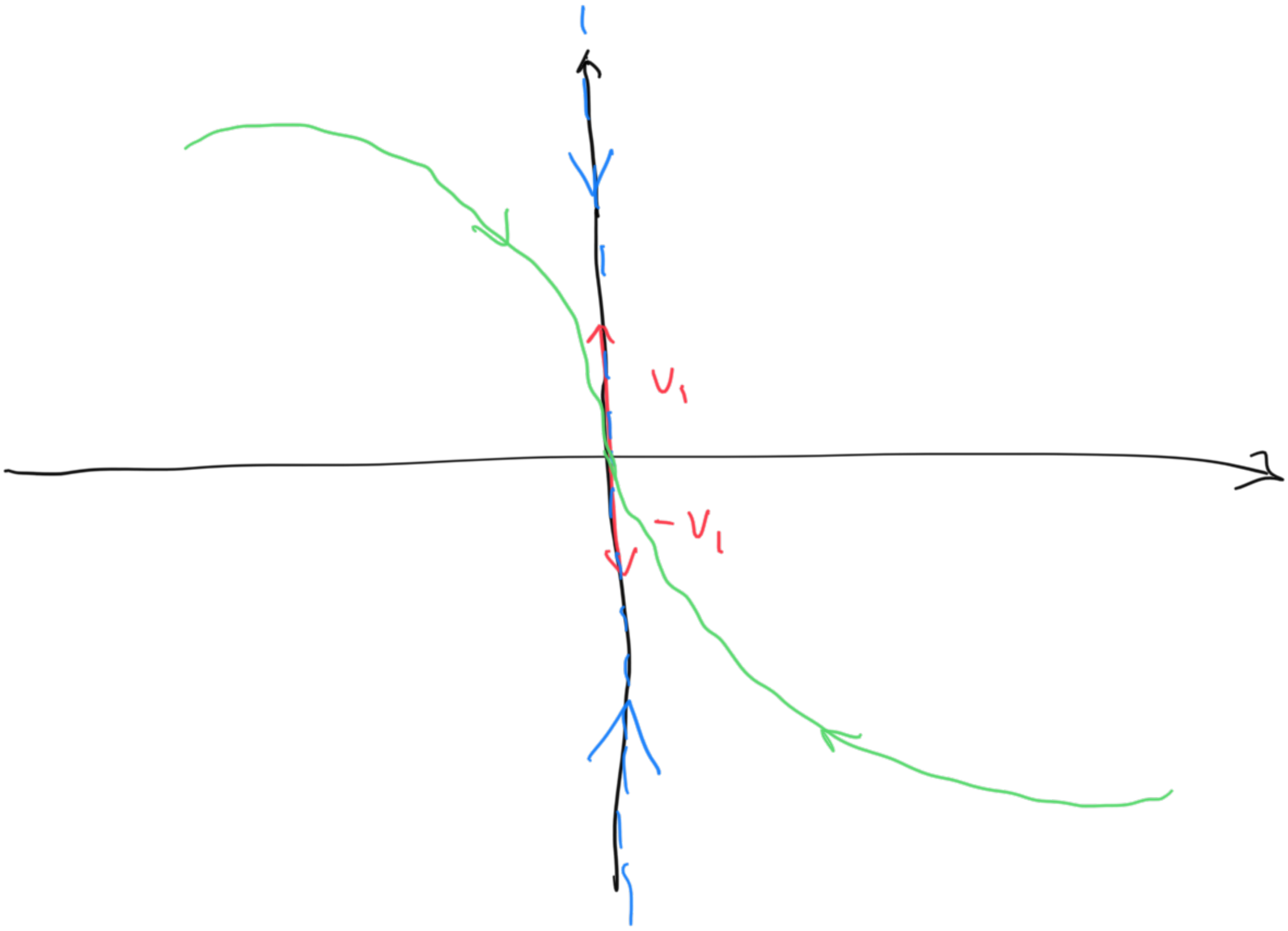
$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$c = 1$$

$$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$y(t) = C_1 \underline{e^{-t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}} + \underline{C_2 e^{-t}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)}$$



$$a = 0, \quad A = \begin{pmatrix} a & a^2+a \\ 1 & a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$p(\lambda) = \lambda^2$$

$$Av = 0$$

$$Y' = AY$$

$$0 = 0$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ a \end{pmatrix} = 0$$

$$a = 0$$

$$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(A - \lambda I)\, w = v$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$c = 1$$

$$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

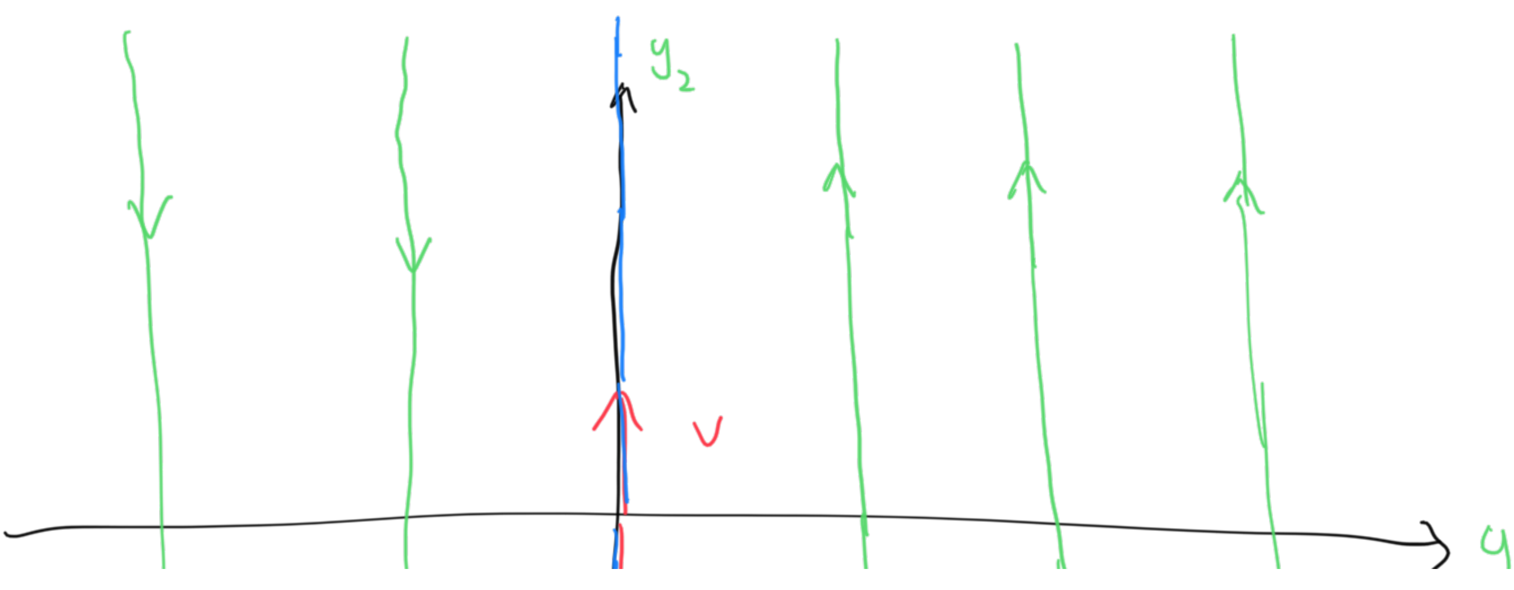
$$y(t) = \underline{C_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}} + C_2 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

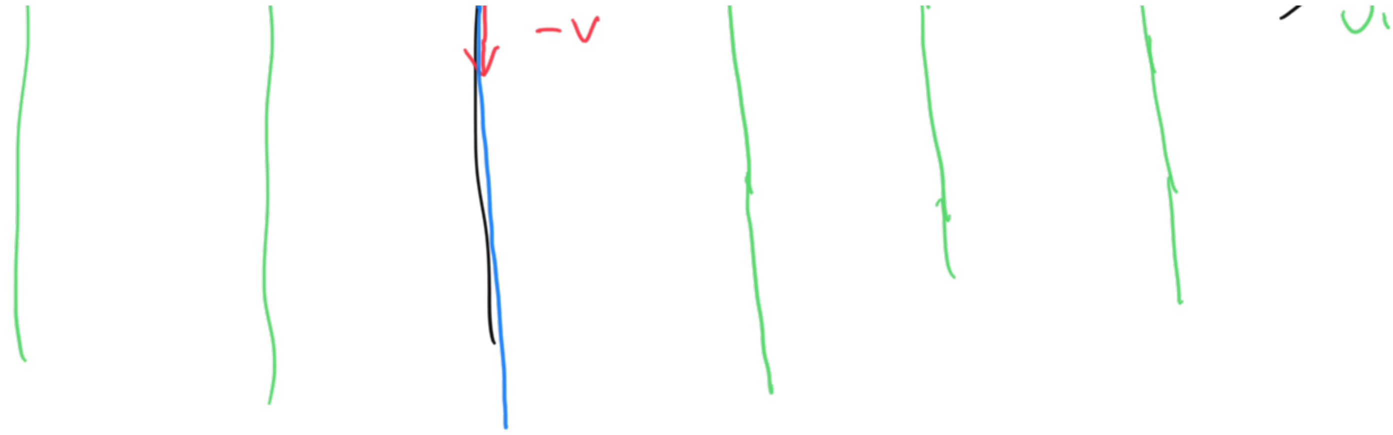
$$= \begin{pmatrix} 0 \\ C_1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ t \end{pmatrix}$$

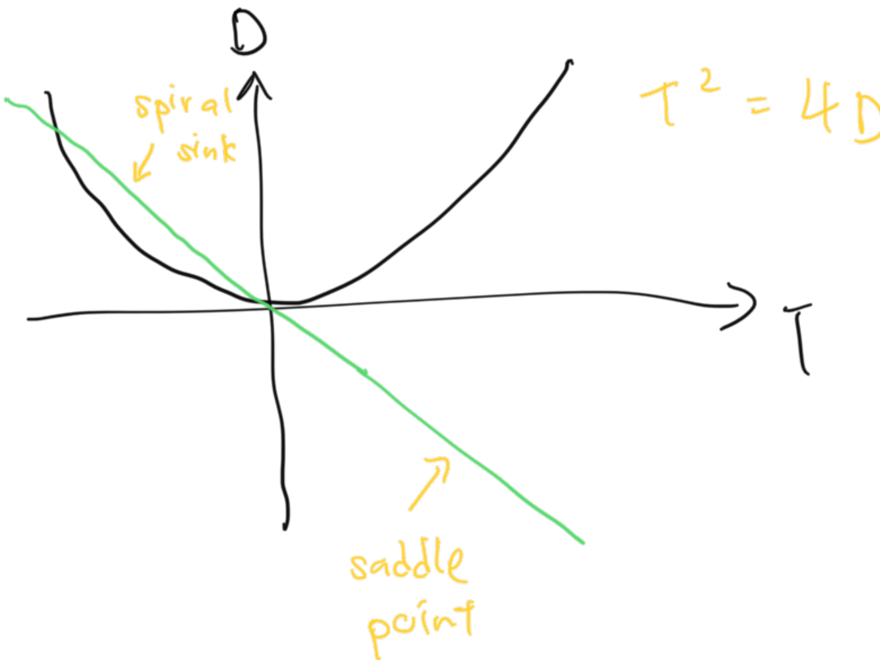
$$= \begin{pmatrix} C_2 \\ C_2 t + C_1 \end{pmatrix}$$

$$y_1 = C_2$$

$$y_2 = C_2 t + C_1$$







$T^2 = 4D$ $\quad$ $T^2 - 4D < 0$

$T^2 - 4D > 0$

two distinct real solutions

$$p(\lambda) = \lambda^2 - T\lambda + D$$

$\lambda_1 \quad \lambda_2$

$$D = \lambda_1 \lambda_2 \neq 0$$

$$D < 0$$

$\lambda_1 \quad \lambda_2$

$$D = \lambda_1 \lambda_2 > 0$$

$$T = \lambda_1 + \lambda_2$$

two distinct complex solutions

$$\lambda = \frac{T \pm \sqrt{T^2 - 4D}}{2}$$