Math 33B : 7/5

Homework 1 , Question 2

2.(a) $80°F$, 2pm

$64°F$, 2am

$$A(t) = C \sin\left(\frac{\pi t}{12}\right) + D$$

$$A(14) = 80$$

$$C \sin\left(\frac{\pi}{12} \cdot 14\right) + D = 80$$

$$C\left(-\frac{1}{2}\right) + D = 80 \quad \text{——— (1)}$$

$$A(2) = 64$$

$$C \sin\left(\frac{\pi}{12} \cdot 2\right) + D = 64$$

$$C\left(\frac{1}{2}\right) + D = 64 \quad \text{——— (2)}$$

$$D = 72 \ , \quad C = -16$$

$$A(t) = -16 \sin\left(\frac{\pi}{12} t\right) + 72$$

2(b) $T(t)$, $T(0) = 50°F$

$$\frac{dT}{dt} = -0.03\,(T - A(t))$$

$$\frac{dT}{dt} + \frac{3}{100} T = \frac{3}{100}\left(-16 \sin\left(\frac{\pi}{12} t\right) + 72\right)$$

$$\frac{dT}{\ } + \frac{3}{\ } T = -\frac{12}{\ } \sin\left(\frac{\pi t}{\ }\right) + \frac{54}{\ }$$

$$\frac{dT}{dt} + \frac{3}{100} \quad \frac{12}{25} \sin\left(\frac{\pi t}{12}\right) \quad \frac{54}{25}$$

Let $\mu(t) = e^{\int \frac{3}{100} dt} = e^{\frac{3}{100} t}$

$$(\mu T)' = -\frac{12}{25} e^{\frac{3}{100}t} \sin\left(\frac{\pi t}{12}\right) + \frac{54}{25} e^{\frac{3}{100}t}$$

Question $\int e^{at} \sin bt \, dt = ?$

$$\int e^{at} \sin bt \, dt$$

$\sin bt$	$\frac{1}{a} e^{at}$
$b \cos bt$	e^{at}

$$= \frac{1}{a} e^{at} \sin bt - \frac{b}{a} \int e^{at} \cos bt \, dt$$

$\cos bt$	$\frac{1}{a} e^{at}$
$-b \sin bt$	e^{at}

$$= \frac{1}{a} e^{at} \sin bt - \frac{b}{a^2} e^{at} \cos bt - \frac{b^2}{a^2} \int e^{at} \sin bt \, dt$$

$$\int e^{at} \sin bt \, dt = \frac{1}{a} e^{at} \sin bt - \frac{b}{a^2} e^{at} \cos bt - \frac{b^2}{a^2} \int e^{at} \sin bt \, dt$$

$$\left(1 + \frac{b^2}{a^2}\right) \int e^{at} \sin bt \, dt = \frac{1}{a} e^{at} \sin bt - \frac{b}{a^2} e^{at} \cos bt$$

$$\int e^{at} \sin bt \, dt = \frac{a}{a^2+b^2} e^{at} \sin bt - \frac{b}{a^2+b^2} e^{at} \cos bt$$

Check $\frac{d}{dt} \left(\frac{a}{a^2+b^2} e^{at} \sin bt - \frac{b}{a^2+b^2} e^{at} \cos bt \right)$

$$= \frac{a}{a^2+b^2} \left(a e^{at} \sin bt + b e^{at} \cos bt \right)$$

$$- \frac{b}{a^2+b^2} \left(a e^{at} \cos bt - b e^{at} \sin bt \right)$$

$$= \left(\frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} \right) e^{at} \sin bt + \left(\frac{ab}{a^2+b^2} - \frac{ab}{a^2+b^2} \right) e^{at} \cos bt$$

$$= e^{at} \sin bt$$

Alternatively,

$$e^{x+iy} = e^x e^{iy}$$
$$= e^x(\cos y + i\sin y)$$

$$\int e^{(a+bi)t}\,dt = \frac{1}{a+bi}e^{(a+bi)t}$$

$$\int (e^{at}\cos bt + i\,e^{at}\sin bt)\,dt = \frac{1}{a+bi}e^{(a+bi)t}$$

$$\underline{\int e^{at}\cos bt\,dt} + \underline{i\int e^{at}\sin bt\,dt}$$

$$= \frac{a-bi}{a^2+b^2}\left(e^{at}\cos bt + i\,e^{at}\sin bt\right)$$

$$= \underline{\frac{a}{a^2+b^2}e^{at}\cos bt + \frac{b}{a^2+b^2}e^{at}\sin bt}$$
$$+\, i\,\underline{\left(\frac{a}{a^2+b^2}e^{at}\sin bt - \frac{b}{a^2+b^2}e^{at}\cos bt\right)}$$

$$\int e^{at}\sin bt\,dt = \frac{a}{a^2+b^2}e^{at}\sin bt - \frac{b}{a^2+b^2}e^{at}\cos bt$$
$$\int e^{at}\cos bt\,dt = \frac{a}{a^2+b^2}e^{at}\cos bt + \frac{b}{a^2+b^2}e^{at}\sin bt$$

$$\left(e^{\frac{3}{100}t}T\right)' = -\frac{12}{25}e^{\frac{3}{100}t}\sin\frac{\pi t}{12} + \frac{54}{25}e^{\frac{3}{100}t}$$

$$e^{\frac{3}{100}t}T = -\frac{12}{25}\left(\frac{\frac{3}{100}}{\left(\frac{3}{100}\right)^2+\left(\frac{\pi}{12}\right)^2}e^{\frac{3}{100}t}\sin\frac{\pi t}{12} - \frac{\frac{\pi}{12}}{\left(\frac{3}{100}\right)^2+\left(\frac{\pi}{12}\right)^2}e^{\frac{3}{100}t}\cos\frac{\pi t}{12}\right)$$
$$+ \frac{54}{25}\cdot\frac{100}{3}e^{\frac{3}{100}t} + C$$

$$T(t) = -\frac{12}{25}\left(\frac{\frac{3}{100}}{\quad}\sin\left(\frac{\pi t}{12}\right) - \frac{\frac{\pi}{12}}{\quad}\cos\left(\frac{\pi t}{12}\right)\right) + 72 + Ce^{-\frac{3}{100}t}$$

$$T(t) = -\frac{\cdots}{25}\left(\frac{\cdots}{(\frac{3}{100})^2+(\frac{\pi}{12})^2}\sin\left(\frac{\cdots}{12}\right) - \frac{\cdots}{(\frac{3}{100})^2+(\frac{\pi}{12})^2}\cos\left(\frac{\cdots}{12}\right)\right) + 12 + Ce^{\cdots}$$

$$T(0) = 50°F \Rightarrow C = \ldots$$

$$\lim_{t\to\infty} Ce^{-\frac{3}{100}t} = 0$$

$$A(t) = C\sin\left(\frac{\pi t}{12} + \theta\right) + D$$

$$\int e^{at}\sin bt$$

$$\int e^{at}\sin(b(t-c))\,dt$$

Let $t = u + c$

$dt = du$

$$= \int e^{a(u+c)}\sin bu\,du$$

$$= e^{ac}\int e^{au}\sin bu\,du$$