Math 33B: 7/5

Homework 1 Question 2

2.(a) $80°F$ 2pm

$64°F$ 2am

$$A(t) = C\sin\left(\frac{\pi}{12}t\right) + D$$

$$A(14) = 80 = C\sin\left(\frac{\pi}{12}\cdot 14\right) + D$$

$$80 = C\left(-\frac{1}{2}\right) + D \qquad \text{——— } (1)$$

$$A(2) = 64 = C\sin\left(\frac{\pi}{12}\cdot 2\right) + D$$

$$64 = C\left(\frac{1}{2}\right) + D \qquad \text{——— } (2)$$

$$D = 72, \quad C = -16$$

$$A(t) = -16\sin\left(\frac{\pi}{12}t\right) + 72$$

2.(b) $T(t)$, $T(0) = 50°F$

$$\frac{dT}{dt} = -0.03\,(T - A(t))$$

$$\frac{dT}{dt} + \frac{3}{100}T = \frac{3}{100}\left(-16\sin\left(\frac{\pi}{12}t\right) + 72\right)$$

Let $\mu(t) = e^{\int \frac{3}{100}dt} = e^{\frac{3}{100}t}$

$$(\mu T)' = \ldots 12\, e^{\frac{3}{100}t} \sin\left(\frac{\pi t}{\ldots}\right) + 54\, e^{\frac{3}{100}t}$$

$(u1) = -\frac{}{25} e \quad \sin(12) + \frac{}{25} e$

Question: $\int e^{at} \sin bt \, dt = ?$

$$\int e^{at} \sin bt \, dt$$

$\sin bt \quad \frac{1}{a} e^{at}$

$b \cos bt \quad e^{at}$

$$= \frac{1}{a} e^{at} \sin bt - \frac{b}{a} \int e^{at} \cos bt \, dt$$

$\cos bt \quad \frac{1}{a} e^{at}$

$-b \sin bt \quad e^{at}$

$$= \frac{1}{a} e^{at} \sin bt - \frac{b}{a^2} e^{at} \cos bt - \frac{b^2}{a^2} \int e^{at} \sin bt \, dt$$

$$\int e^{at} \sin bt \, dt = \frac{1}{a} e^{at} \sin bt - \frac{b}{a^2} e^{at} \cos bt - \frac{b^2}{a^2} \int e^{at} \sin bt \, dt$$

$$\left(1 + \frac{b^2}{a^2}\right) \int e^{at} \sin bt \, dt = \frac{1}{a} e^{at} \sin bt - \frac{b}{a^2} e^{at} \cos bt$$

$$\int e^{at} \sin bt \, dt = \frac{a}{a^2+b^2} e^{at} \sin bt - \frac{b}{a^2+b^2} e^{at} \cos bt$$

Check $\frac{d}{dt}\left(\frac{a}{a^2+b^2} e^{at} \sin bt - \frac{b}{a^2+b^2} e^{at} \cos bt \right)$

$$= \frac{a}{a^2+b^2}\left(a e^{at} \sin bt + b e^{at} \cos bt \right)$$

$$- \frac{b}{a^2+b^2}\left(a e^{at} \cos bt - b e^{at} \sin bt \right)$$

$$= \left(\frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} \right) e^{at} \sin bt + \left(\frac{ab}{a^2-b^2} - \frac{ab}{a^2+b^2} \right) e^{at} \cos bt$$

$$= e^{at} \sin bt$$

Alternatively,

$$e^{x+iy} = e^x \cdot e^{iy}$$
$$= e^x (\cos y + i \sin y)$$

$$\int e^{(a+bi)t}\,dt = \frac{1}{a+bi} e^{(a+bi)t}$$

$$\int (e^{at}\cos bt + i\, e^{at}\sin bt)\,dt = \frac{1}{a+bi} e^{(a+bi)t}$$

$$\int e^{at}\cos bt\,dt + i \underline{\int e^{at}\sin bt\,dt} =$$

$$\frac{a-bi}{a^2+b^2}\left(e^{at}\cos bt + i\, e^{at}\sin bt\right)$$

$$= \frac{a}{a^2+b^2} e^{at}\cos bt + \frac{b}{a^2+b^2} e^{at}\sin bt$$
$$+ i \underline{\left(\frac{a}{a^2+b^2} e^{at}\sin bt - \frac{b}{a^2+b^2} e^{at}\cos bt\right)}$$

$$\int e^{at}\sin bt\,dt = \frac{a}{a^2+b^2} e^{at}\sin bt - \frac{b}{a^2+b^2} e^{at}\cos bt$$
$$\int e^{at}\cos bt\,dt = \frac{a}{a^2+b^2} e^{at}\cos bt + \frac{b}{a^2+b^2} e^{at}\sin bt$$

$$\left(e^{\frac{3}{100}t}\, T\right)' = -\frac{12}{25} e^{\frac{3}{100}t} \sin\left(\frac{\pi t}{12}\right) + \frac{54}{25} e^{\frac{3}{100}t}$$

$$e^{\frac{3}{100}t}\, T = -\frac{12}{25}\left(\frac{\frac{3}{100}}{\left(\frac{3}{100}\right)^2 + \left(\frac{\pi}{12}\right)^2} e^{\frac{3}{100}t} \sin\left(\frac{\pi t}{12}\right) - \frac{\frac{\pi}{12}}{\left(\frac{3}{100}\right)^2 + \left(\frac{\pi}{12}\right)^2} e^{\frac{3}{100}t} \cos\left(\frac{\pi t}{12}\right)\right)$$
$$+ \frac{54}{25} \cdot \frac{100}{3} e^{\frac{3}{100}t} + C$$

$$T(t) = -\frac{12}{25}\left(\frac{\frac{3}{100}}{\left(\frac{3}{100}\right)^2 + \left(\frac{\pi}{12}\right)^2} \underline{\sin\left(\frac{\pi t}{12}\right)} - \frac{\frac{\pi}{12}}{\left(\frac{3}{100}\right)^2 + \left(\frac{\pi}{12}\right)^2} \underline{\cos\left(\frac{\pi t}{12}\right)}\right) + 72 + \underline{Ce^{-\frac{3}{100}t}}$$

$T(0) = 50$

$$\lim_{t \to \infty} C e^{-\frac{3}{100}t} = 0$$

$$\int e^{at} \sin(bt + C)\, dt = \int \frac{1}{b} e^{\frac{a}{b}(u-c)} \sin u \, du$$

Let $t = \frac{1}{b}(u - C)$ $$= \frac{e^{-\frac{ac}{b}}}{b} \int e^{\frac{a}{b}u} \sin u \, du$$