Math 33B: 6/30

1. $$\frac{1}{y-y^2}\, y' = te^t$$

$$\frac{1}{y-y^2}\, dy = te^t\, dt$$

$$\int \frac{1}{y-y^2}\, dy = \int te^t\, dt$$

$$\int \frac{1}{y-y^2}\, dy = \int \left(\frac{1}{y} + \frac{1}{1-y}\right) dy = \ln|y| - \ln|1-y|$$

$$\frac{1}{y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}$$

$$1 = A(1-y) + By$$

Let $y=1$, then $1 = B \cdot 1$

$$B = 1$$

Let $y=0$, then $1 = A \cdot 1 + B \cdot 0$

$$A = 1$$

$$\frac{1}{y-y^2} = \frac{1}{y} + \frac{1}{1-y}$$

$$\int te^t\, dt = te^t - \int e^t\, dt \qquad \begin{array}{cc} t & e^t \\ 1 & e^t \end{array}$$

$$= te^t - e^t$$

$$\ln|y| - \ln|1-y| = te^t - e^t + C$$

$$\frac{y}{1-y} = C_1 e^{te^t - e^t}$$

$$y = \frac{C_1 e^{te^t - e^t}}{1 + C_1 e^{te^t - e^t}}$$

Sec 2.2 14. $y' = \dfrac{-2t(1+y^2)}{y}$, $y(0) = 1$

$$\frac{y}{1+y^2}\, dy = -2t\, dt$$

$$\int \frac{y}{1+y^2}\, dy = \int -2t\, dt$$

$$\frac{1}{2}\ln(1+y^2) = -t^2 + C$$

$$1 + y^2 = C_1 e^{-2t^2}$$

$y(0) = 1$, $\quad 2 = C_1 \cdot 1 \quad \Rightarrow \quad C_1 = 2$

$$y(t) = \pm\sqrt{2e^{-2t^2} - 1}$$

$y(0) = 1$, $\quad \pm\sqrt{2 \cdot 1 - 1} = \pm\sqrt{1} = \pm 1$

$$y(t) = \sqrt{2e^{-2t^2} - 1}$$

$$2e^{-2t^2} - 1 > 0$$

$${}^{-2t^2} \qquad 1$$

$$e \quad > \frac{1}{2}$$

$$-2t^2 > \ln\frac{1}{2}$$

$$t^2 < \frac{1}{2}\ln 2$$

$$-\sqrt{\frac{1}{2}\ln 2} < t < \sqrt{\frac{1}{2}\ln 2}$$

$$f(x) = \sqrt{x}, \quad x \geq 0$$

$$f'(x) = \frac{1}{2\sqrt{x}}, \quad x > 0$$

Sec 2.4   40.
$$\begin{cases} x' - \frac{2}{t^2}x = \frac{1}{t^2} \\ x(1) = 0 \end{cases}$$

$$x' = \frac{2}{t^2}x + \frac{1}{t^2}$$

Consider $\quad x' = \frac{2}{t^2}x$

$$\frac{1}{x}\,dx = \frac{2}{t^2}\,dt$$

$$\ln x = -\frac{2}{t}$$

$$x(t) = e^{-\frac{2}{t}}$$

$$\ln|x| = -\frac{2}{t} + C$$

$$x(t) = C_1 e^{-\frac{2}{t}}$$

Let $\quad x(t) = v(t)\, e^{-\frac{2}{t}}$

$$\left(ve^{-\frac{2}{t}}\right)' = \frac{2}{t^2}\left(ve^{-\frac{2}{t}}\right) + \frac{1}{t^2}$$

$$v' e^{-\frac{2}{t}} + v\left(e^{-\frac{2}{t}}\right)' = \frac{2}{t^2}\left(v e^{-\frac{2}{t}}\right) + \frac{1}{t^2}$$

$$v' e^{-\frac{2}{t}} + v \frac{2}{t^2} e^{-\frac{2}{t}} = \frac{2}{t^2}\left(v e^{-\frac{2}{t}}\right) + \frac{1}{t^2}$$

$$v' e^{-\frac{2}{t}} = \frac{1}{t^2}$$

$$v' = \frac{e^{\frac{2}{t}}}{t^2}$$

$$e^{\frac{2}{t}} \left(2 \cdot \left(-\frac{1}{t^2}\right)\right)$$

$$v(t) = \int \underline{\frac{e^{\frac{2}{t}}}{t^2}} \, dt$$

$$= \underline{-\frac{1}{2} e^{\frac{2}{t}}} + C$$

$$\frac{d}{dt} e^{\frac{2}{t}} = e^{\frac{2}{t}} \cdot \left(2 \cdot \left(-\frac{1}{t^2}\right)\right)$$

$$= -2 \frac{e^{\frac{2}{t}}}{t^2}$$

$$\frac{d}{dt}\left(-\frac{1}{2} e^{\frac{2}{t}}\right) = \frac{e^{\frac{2}{t}}}{t^2}$$

$$x(t) = v \cdot e^{-\frac{2}{t}}$$

$$= -\frac{1}{2} + C e^{-\frac{2}{t}}$$

$$x(1) = 0$$

$$-\frac{1}{2} + C e^{-2} = 0$$

$$C = \frac{e^2}{2}$$

$$x(t) = -\frac{1}{2} + \frac{e^2}{2} e^{-\frac{2}{t}}$$

$$X(t) = -\frac{1}{2} + \frac{e^2}{2} e$$

Alternatively,

$$\text{Let } \mu(t) = e^{\int -\frac{2}{t^2} dt}$$

$$= e^{\frac{2}{t}}$$

$$\mu\left(x' - \frac{2}{t^2} x\right) = \mu \frac{1}{t^2}$$

$$(\mu x)' = \mu \frac{1}{t^2}$$

$$\left(e^{\frac{2}{t}} x\right)' = e^{\frac{2}{t}} \frac{1}{t^2}$$

$$e^{\frac{2}{t}} x = \int \frac{e^{\frac{2}{t}}}{t^2} dt$$

$$e^{\frac{2}{t}} x = -\frac{1}{2} e^{\frac{2}{t}} + C$$

$$x(t) = -\frac{1}{2} + C e^{-\frac{2}{t}}$$

$$x(t) = -\frac{1}{2} + \frac{e^2}{2} e^{-\frac{2}{t}}$$

$$y'' + p y' + q y = r$$

$$y'' + p y' + q y = 0$$