Math 33B : 6/30

1. $\frac{1}{y-y^2} y' = t e^t$

$$\frac{1}{y-y^2} dy = t e^t dt$$

$$\int \frac{1}{y-y^2} dy = \int t e^t dt$$

$$\int \frac{1}{y-y^2} dy = \int \left(\frac{1}{y} + \frac{1}{1-y}\right) dy = \ln|y| - \ln|1-y|$$

$$\frac{1}{y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}$$

$$1 = A(1-y) + By$$

Let $y=1$, then $1 = B \cdot 1$

$$B = 1$$

Let $y=0$, then $1 = A + B \cdot 0$

$$A = 1$$

$$\frac{1}{y-y^2} = \frac{1}{y} + \frac{1}{1-y}$$

$$\int t e^t dt = te^t - \int e^t dt$$

t	e^t
1	e^t

$$= te^t - e^t$$

$$\ln|y| - \ln|1-y| = te^t - e^t + C$$

$$\frac{y}{1-y} = C_1 e^{te^t - e^t}$$

$$y = \frac{C_1 e^{te^t - e^t}}{1 + C_1 e^{te^t - e^t}}$$

Sec 2.2 14. $y' = \frac{-2t(1+y^2)}{y}$, $y(0) = 1$

$$\frac{y}{1+y^2} dy = -2t \, dt$$

$$\int \frac{y}{1+y^2} dy = \int -2t \, dt$$

$$\frac{1}{2}\ln(1+y^2) = -t^2 + C$$

$$1 + y^2 = C_1 e^{-2t^2}$$

$y(0) = 1$, $2 = C_1 \cdot 1 \Rightarrow C_1 = 2$

$$y = \pm\sqrt{2e^{-2t^2} - 1} \qquad y = \sqrt{2e^{-2t^2} - 1}$$

$$2e^{-2t^2} - 1 > 0$$

$$e^{-2t^2} > \frac{1}{2}$$

$$-2t^2 > \ln\frac{1}{2}$$

$$2t^2 < \ln 2$$

$$t^2 < \frac{1}{2}\ln 2$$

$$-\sqrt{\frac{1}{2}\ln 2} < t < \sqrt{\frac{1}{2}\ln 2}$$

$f(x) = \sqrt{x} \qquad x \geqslant 0$

$f'(x) = \frac{1}{2\sqrt{x}} \qquad x > 0$

$y' = \sqrt{y} \qquad y(0) = 1$

$$\frac{1}{\sqrt{y}}\, dy = dt$$

$$2\sqrt{y} = t + C$$

$$y = \frac{1}{4}(t+C)^2$$

$1 = \frac{1}{4}C^2 \quad , \quad C = \pm 2$

$y = \frac{1}{4}(t+2)^2 \quad , \quad y = \frac{1}{4}(t-2)^2$

$$y' = \frac{1}{2}(t+2)$$

$$\sqrt{y} = \sqrt{\frac{1}{4}(t+2)^2} = \frac{1}{2}|t+2|$$

$t > -2$

$$y' = \frac{1}{2}(t-2)$$

$$\sqrt{y} = \sqrt{\frac{1}{4}(t-2)^2} = \frac{1}{2}|t-2|$$

$$t \geq 2 \qquad (2, \infty)$$

$[a, b)$

Sec 2.4 40. $x' - \frac{2}{t^2}x = \frac{1}{t^2}$, $x(1) = 0$

$$x' = \frac{2}{t^2}x + \frac{1}{t^2}$$

Consider $x' = \frac{2}{t^2}x$

$$\frac{1}{x}\,dx = \frac{2}{t^2}\,dt$$

$$\ln|x| = -\frac{2}{t}$$

$$x(t) = e^{-\frac{2}{t}}$$

Let $x(t) = v(t)\,e^{-\frac{2}{t}}$

$$\left(ve^{-\frac{2}{t}}\right)' = \frac{2}{t^2}\left(ve^{-\frac{2}{t}}\right) + \frac{1}{t^2}$$

$$v'e^{-\frac{2}{t}} + \cancel{v\underbrace{\left(e^{-\frac{2}{t}}\right)'}_{\frac{2}{t}\cdot e^{-\frac{2}{t}}}} = \cancel{\frac{2}{t^2}ve^{-\frac{2}{t}}} + \frac{1}{t^2}$$

$$e^{\frac{2}{t}}$$

$$v = \frac{\text{[illegible]}}{t^2}$$

$$v(t) = -\frac{1}{2} e^{\frac{2}{t}} + C$$

$$x(t) = -\frac{1}{2} + C e^{-\frac{2}{t}}$$