Math 33B
Homework 4
Due Sunday, July 24, 2022

Do the following problems from each section of the textbook:

- 9.4: 2,4,10,11,18,22. (Also find the general solution to the system if the problem doesn't ask for it. You may ignore any parts about using numerical solvers.)
- 10.1: 2,4,6
- 10.2: 6,12
- 10.3: 10,14

Do the following additional problems:

1. Consider the one parameter family of linear systems

$$Y' = \begin{pmatrix} a & a^2+a \\ 1 & a \end{pmatrix} Y$$

   (a) Sketch the curve in the trace-determinant plane that comes from varying the parameter a.

   (b) Determine all bifurcation values of a and describe the different types of phase portraits that are exhibited by this one-parameter family.

2. Consider the differential equation $y'' - y + y^3 = 0$. (This models the motion of a certain spring that doesn't obey Hooke's law.)

   (a) Convert the differential equation into a 2D system of the form $y' = f(y, v)$ and $v' = g(y, v)$ for some functions f, g. Find the equilibrium points, and classify their behavior.

   (b) Find a function $H(y, v)$ such that $\partial H/\partial v = f(y, v)$ and $-\partial H/\partial y = g(y, v)$. (If you have taken multivariable calculus, then you should know this will then tell you the level curves of H are the trajectories of solutions.)

   (c) Use the phase portrait applet on the BruinLearn page and examine the timeplot of the solution curve passing through $(1, 1)$. Explain how you can obtain this graph from knowing the phase portrait.

Optional challenge (do not turn in): Consider the two parameter family of linear systems

$$Y' = \begin{pmatrix} a & 1 \\ b & 1 \end{pmatrix} Y.$$

In the ab-plane, identify all regions where this system possesses a saddle, sink, center, etc. *(Hint: mimic the idea of the trace-determinant plane, but in the ab-plane instead.)*