Math 33B
Homework 3
Due Sunday, July 17, 2022

Do the following problems from each section of the textbook:

- 4.5: 20,28,38,40
- 4.6: 4,10,14
- 9.1: 10,18
- 9.2: 10,20,30

Do the following additional problems:

1. Consider the IVP $y' = t - y, \quad y(0) = 1$.
   (a) Use Euler's method with step sizes $h = 1, .5, .25, .125$ to approximate $y(1)$ (you should probably use a calculator for this!).
   (b) Find an explicit solution to the IVP, and compute the error in your approximation for each value of h you used. How does the error change each time you cut h in half?
2. For this problem you'll want to use an online applet like https://www.geogebra.org/m/NUeFjm9J to graph numerical approximations using Euler's method.
   (a) Consider the IVP $y' = 12y(4 - y), \quad y(0) = 1$. Perform a qualitative analysis of this differential equation using the techniques of chapter 2 to give a sketch of the solution $y(t)$. Graph the approximate solution in the applet using $h = .2, .1, .05$. Describe what you see.
   (b) Repeat the above for $y' = -5y, \quad y(0) = 1$ with $h = 1, .75, .5, .25$.
   (c) Finally, do the same for $y' = (y - 1)^2, \quad y(0) = 0$ with $h = 1.25, 1, .5, .25$.
   (d) Play around with the applet to your heart's desire using whatever other examples you choose. Summarize whatever other "disasters" you may run into. How does this experiment make you feel about Euler's method?
3. Consider the IVP $y'' - (1 - y^2)y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1$.
   (a) Use the method outlined in class to convert the second order differential equation into a system of first order differential equations.
   (b) Use Euler's method with step size $h = .1$ to approximate $y(1)$.