Midterm Guide

We've covered a lot of content in the first half of the course! Here is a list of what I expect you should be able to do:

Qualitative techniques

- You should understand what a slope field is, how to draw them, and how to sketch a solution curve to an initial value problem given a slope field.
- You should know what it means for a differential equation to be autonomous, and should be able to identify if a given differential equation (via an explicit formula or a slope field) is autonomous or not.
- You should understand our qualitative techniques for analyzing autonomous differential equations: this means phase diagrams, stability analysis, sketching solution curves without formulas, and bifurcation.

Analytic techniques

- You should be comfortable with the basics of calculus. This means taking derivatives (and partial derivatives!), and computing basic integrals using techniques like substitution, integration by parts, and partial fractions.
- Given a first order differential equation, you should be able to identify if it's linear, non-linear, separable, exact, homogeneous, etc., decide what the appropriate method for solving it is, and carry out the computational details.
- You should understand the idea of an integrating factor and be able to use one to solve first order differential equations. You should be able to determine when a first order differential equation has an integrating factor of a special form (e.g. depending on only one variable).
- Given a homogeneous second order differential equation with a known solution, you should be able to carry out the technique outlined in HW 2 to find the general solution.
- You should be familiar with the methods of variation of parameter and undetermined coefficients and how they can be used to solve differential equations (first/second order).
- Given a second order differential equation with constant coefficients (homogeneous or not), you should be able to determine an appropriate method for solving it, and carry out the computational details.

Theory

- You should understand the statement of the existence and uniqueness theorem for first order differential equations, and be able to apply it. This means being able to check if a given initial value problem will have a unique solution, recognize when it's possible for an IVP to have multiple solutions, and give bounds on solutions curves.

- You should understand how to prove that two functions are linearly independent.
- You should know the definition of the Wronskian and what it says about linear independence.
- You should understand the structure of the general solution to an inhomogeneous differential equation (both first/second order).

Applications

- You should be comfortable with the three different applications of differential equations that we've talked about so far: mixing problems, Newton's law of cooling, and the logistic model for population growth.