**Math 33B**
**Differential Equations**

**Midterm**

**Directions:** Do the problems below. You have 24 hours to complete this exam, from 8:00 AM PST on Tuesday, July 12th to 8:00 AM PST on Wednesday, July 13th, by which time you must scan and upload your exam on Gradescope. You may use any course resources from our BruinLearn page, the textbook, or your notes, but you may not use other internet resources, nor may you discuss the exam with anyone other than me. Do not use methods that have not been covered in class. You may use a basic calculator. Show your work. Write full sentences when necessary.

Name: _____

UID: _____

| Question | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 10 | |
| 3 | 15 | |
| 4 | 13 | |
| Total: | 50 | |

1. (12 pts.) Find non-zero real numbers $a, b$ that make $\mu(x, y) = x^a e^{by}$ an integrating factor for the differential equation $(x^2 + e^{-y})\,dx + (x^3 + x^2 y)\,dy = 0$, and use your integrating factor to find the general solution.

2. (10 pts.) Suppose that $y(t)$ is a solution to the initial value problem

$$\frac{dy}{dt} = \frac{y-t}{y^2+1} + 1, \quad y(0) = 1.$$

Is it possible that $y(1) = 0$? Justify your answer.

3. (15 pts.)

   (a) (10 pts.) Find the general solution to each differential equation.

   - $y'' - 2y' + y = te^t$
   - $t^2y'' - ty' + y = 0$, given that $y_2(t) = t$ is one solution.

   (b) (5 pts.) Does there exist a differential equation of the form $y'' + p(t)y' + q(t)y = 0$ with fundamental solutions $y_1(t) = t$ and $y_2(t) = t^2 + 1$ on the interval $(-2, 2)$? Justify your answer.

4. (13 pts.) Suppose that you know the population $P(t)$ ($t$ measured in years) of deer in a forest can be modeled according to the differential equation

$$\frac{dP}{dt} = P\left(1 - \frac{P}{125}\right)\left(\frac{P}{25} - 1\right)$$

   (a) (3 pts.) Draw the phase diagram for the differential equation and classify the stability of each equilibrium point. Sketch the graph of at least one solution between each pair of adjacent equilibrium solutions as well as above and below each equilibrium solution.

   Currently, there are around 175 deer in the forest and this is starting to cause trouble for the local farmers because the deer are coming into town and eating up all of their crops! To fix this problem, hunters have decided to start hunting some of the deer.

   (b) (5 pts.) Suppose that each year, $h\%$ of the deer population is allowed to be hunted. What is the maximal percentage of deer that can be safely hunted each year without risk of the population dying out? Justify your answer.

   (c) (5 pts.) What is the maximum percentage of deer that are allowed to be hunted each year if we do not wish the deer population to drop below 100? Justify your answer.