Final Review Guide

Here's a list of what I expect that you should be able to do for the final exam.

Past content

- Everything that I wrote for the midterm review guide still applies!

Qualitative techniques

- You should know what a phase portrait is and how it can be sketched by hand.
- You should know the different equilibrium types for a linear 2×2 system of differential equations, and how to sketch their phase portraits. In particular, I expect that you should be able to **accurately** sketch the phase portrait for all types except centers and spirals. In those cases, you should be able to determine the rotation direction.
- You should be able to locally sketch the behavior of a 2×2 non-linear system near a generic equilibrium point.
- You should understand how to use nullclines to globally analyze solution trajectories of a non-linear 2×2 system of differential equations.
- Given the phase portrait of a 2×2 system of differential equations, you should be able to roughly describe what graphs of the corresponding functions $x(t)$ and $y(t)$ might look like.
- You should understand the idea of bifurcation for systems of differential equations.

Analytic techniques

- Given a 2×2 linear system of differential equations, you should be able to write down the general solution and solve IVPs. You should be able to compute the exponential of a matrix and find generalized eigenvectors. You should understand how to use these to help you find the general solution to a 3×3 linear system of differential equations.
- Given a non-linear system of differential equations, you should be able to compute the linearization at an equilibrium point.
- You should understand how to convert a second order or higher linear differential equation into a linear system of differential equations. You should also understand how the discussion we had about second order linear differential equations generalizes to third order or higher ones. This means things like the structure of the general solution, the Wronskian, solving constant coefficient higher order ODEs, etc.
- You should understand what the general solution to an inhomogeneous linear system looks like. You should understand what a fundamental matrix is and how to use variation of parameter to solve inhomogeneous linear systems.

Numerical techniques

- You should understand how to use Euler's method and Runge-Kutta to approximate values of solutions to differential equations. You should also understand how these methods work for systems of differential equations, and how they can be applied to differential equations of second order or higher.
- You should understand the relation between the error in the approximation and the step size for each method.

Theory

- You should understand basic linear algebra: how to compute a determinant, how to compute the inverse of a matrix, what an eigenvector/eigenvalue is and how to find them, what trace/determinant are and how they relate to eigenvalues.
- You should understand the trace-determinant plane, and how to derive which regions correspond to which equilibrium types.
- You should understand what it means for an equilibrium point to be generic or not, and when the behavior of a non-linear system is the same as the behavior of the linearization.
- You should understand what it means for an equilibrium point to be stable, unstable, or asymptotically stable, and when the stability of a non-linear system at an equilibrium point is the same as that of the linearization.
- You should understand how to prove an equilibrium point of a 2×2 non-linear system is a center by following the method outlined in lecture and in HW 4.

Applications

- You should understand the difference between a competing species model and a predator-prey model. In particular, you should have an idea of what the solution curves to a predator-prey model look like, and how they can be found.
- Given a competing species or predator-prey model, you should be able to answer contextual questions about the model. Whether or not species will live or die, what population graphs of the species might look like, etc.